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# MAXIMUM STRESSES

IN

# FRAMED BRIDGES,

BY

WILLIAM CAIN,

MEMBER AM. SOC. C. E.,

PROFESSOR OF MATHEMATICS IN THE UNIVERSITY OF NORTH CAROLINA.

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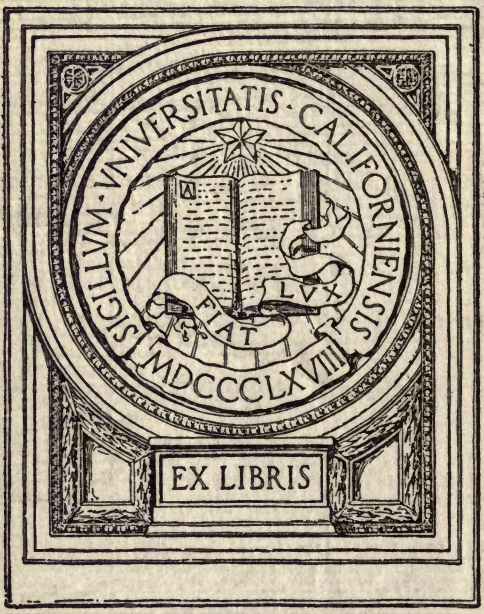


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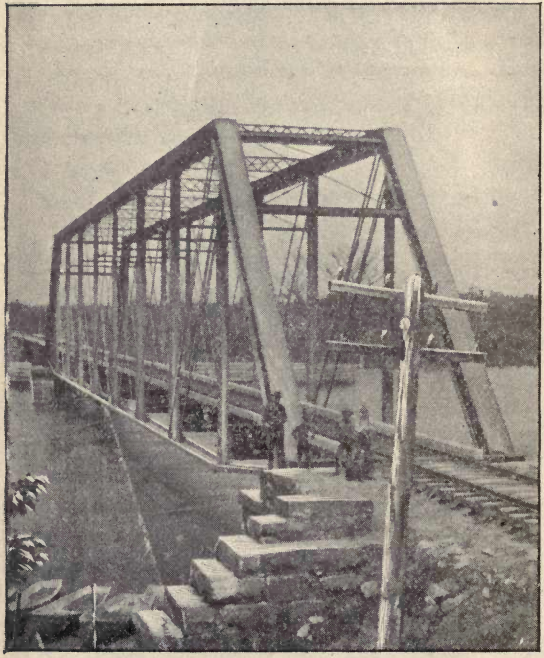

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# PREFACE.

THE first edition of this work was published in Van Nostrand's Magazine for 1878, and was largely concerned with the comparison of the weights of bridges and their most economical depths. These subjects have now been practically solved by bridge engineers, and the result has been the elimination of many types of bridge truss once popular and the retention, by the principle of the "survival of the fittest," of certain leading forms that have proved most economical and otherwise desirable.

From these considerations, it was thought best to confine the present edition to the discussion of those types most used at present, and to leave out any comparison of weights and extended discussions as to minimum material. The work has therefore been entirely rewritten upon a new basis, the aim being to prepare an elementary treatise on the maximum stresses in bridge-

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trusses of selected types, both for uniform and wheel loads, which might serve either as an independent short treatise on the subject or as an introduction to the larger treatises. The subject-matter has been given in more detail perhaps than usual, for experience in teaching has shown that in no other way can a student quickly and surely master its first principles. Attention is especially called to the exact treatment for wheel loads. The aim throughout has been to aid the student by presenting the subject in a simple, clear and, at the same time, thorough manner.

CHAPEL HILL, N. C.,
*April*, 1897.

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# MAXIMUM STRESSES

IN

# FRAMED BRIDGES.

1. A simple truss is a structure composed of straight members, connected at their ends by pins or other means, and designed to transfer loads on it to the abutments at either end, by which it is supported, by the resistances of the members to longitudinal tension or compression.

The ideal truss has the members so connected at their joints that the resultant of the stress on any cross-section of a member will pass through its centre of gravity. This ideal is impossible to attain, except for vertical members, as for all others the weight of the member will cause non-uniform stress on the cross-section. If the lines through the centres of gravity of the cross-sections, or *axes* of members, do not meet in a point at the *joint* where the

members assemble, or if the resultant stress on a member is not axial because it is connected to other members only along a part of its section, or if the members are not free to rotate at a joint under loads, additional stresses are caused over those borne by the members of the ideal truss. Such additional stresses are called secondary, and should always be considered in the final design.

In this little treatise, space will not permit dealing with other than the ideal truss.

2. An important requisite of a well-designed truss is, that the geometric figure formed by the axes of the members should be of invariable form. If we call $m$ the number of bars or members necessary to this end, and $n$ the number of joints or apices, then for a plane figure, assume one side as fixed in position. From the two apices, at its ends, we can fix another apex with two new sides; then another, with two new sides, from *any* two apices previously fixed, and so on; therefore, to each of the $(n-2)$ apices other than the first

two, corresponds two sides, so that the total number of sides, $m$, just necessary to fix the apices, is given by the following relation:

$$m = 2(n - 2) + 1 = 2n - 3 \quad \ldots\ldots (1)$$

If the number of sides exceeds $(2n-3)$ the extra number are called "superfluous." A less number will give a figure that can change its shape without changing the length of its sides.

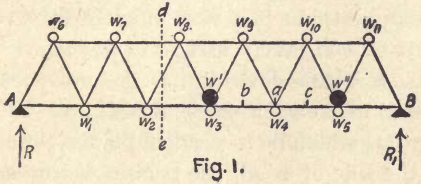


Fig. 1.

As an application, note that in the Warren Truss (fig. 1),* $m = 23$, $n = 13$, so that eq. (1) obtains, and the figure has enough sides to strictly define the form and no more.

In the Queen Post Truss without cross-

* The axes of the members only are shown in this and other figures of trusses.

bracing in the middle panel (fig. 2) $m = 8$ $n = 6$ and eq. (1) is not satisfied, one side being lacking. The stiffness of the chord here

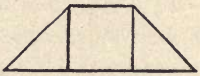

Fig. 2.

is relied on to prevent deformation, so that the truss is an imperfect one from one standpoint, but it is very efficient for a roof truss or a highway bridge in practice.

3. A Framed Bridge is generally composed of two or more trusses, as A *d* B (fig. 1), which lie in vertical planes, parallel to the line of road, the trusses being connected together by *transverse bracing*, as well as by the *floor beams*, which extend from an apex of one truss to a corresponding apex on the other, on which *stringers*, placed parallel to the line of road, rest, which support the cross-ties and rails of a railroad bridge, or the flooring of a highway bridge.

The upper and lower members of a truss

are called the upper and lower *chords*, the bracing between them the *web*.

The upper chord is always in compression, the lower chord in tension, but the web members are generally successively struts and ties. Those web members acting always as ties, are sometimes called *main ties*; those acting always as struts, *main braces* or posts; while those acting under one distribution of the live load as struts, and under another as ties, are called *counter-braces*; the term *counter* now being generally restricted to those web members that are not strained appreciably by the dead load, but are strained when the live load is distributed in a certain manner.

If the roadway is supported by the top chords (of two trusses, say) the bridge is called a *deck* bridge; if by the lower chords it is called a *through* bridge, if there is overhead bracing between the top chords; otherwise it is termed a *pony* truss.

The trussing, in vertical planes, connecting opposite apices of the two trusses of a deck bridge is called *sway* bracing. When

a through bridge is more than 25 feet high, the overhead bracing between trusses in vertical planes is also called sway bracing. The *lateral* bracing lies in horizontal planes, and connects the two top chords together, also the two lower chords. The part of a bridge between two adjacent joints or apices is called a *panel*, a *panel length* being the distance from apex to apex corresponding. In this book, bridges with two trusses will alone be examined.

4. A truss (fig. 1) rests upon abutments at A and B, and is unsupported the distance or *span*, A B. The distance, A B, is the distance between end pins, centre to centre.

The pressures exerted by the truss on the abutments are resisted by their reactions, which will be assumed vertical, as rollers are always placed under one end of a truss over 75 feet span. Bow, in his "Economics of Construction," has given many illustrations of the effects of inclined reactions, due to friction at the abutments resisting expansion or contraction of

chords, whether caused by heat or the passage of loads.

### The Weight of Bridges.

5. The weight of the bridge or *dead load* comprises the weight of the two trusses with all transverse bracing, floor beams, stringers and flooring system, including the planking of a highway bridge, or the cross-ties, rails, guards, spikes, etc., of a railway bridge. Merriman gives the following formula for *the dead load per linear foot* of highway bridges:

$$w = 140 + 12\,b + 0.2\,bl - 0.4\,l \ldots\ldots (2)$$

in which $b$ = width of bridge in feet (including sidewalks, if any), $l$ = span in feet, and $w$ = dead load in pounds per linear foot.

The formula gives weights agreeing fairly well with bridges belonging to class A of Waddel's Specifications, which allows 100 pounds per square foot of flooring for the live load on spans from 0 to 100 feet, and 65 pounds for spans of 350 feet and over, proportionate amounts being allowed for intermediate spans, provided that in no

case shall the live load per linear foot be less than 1800 pounds. In this "Class A," intended for city bridges, a road roller, having 12,000 pounds on the front wheel, 4 feet wide, and 9000 pounds on each rear wheel, 1 foot 8 inches wide, is to be used in computing stresses when this loading gives the maximum stress on members, such as floor beams, stringers, etc. The distance between axles of front and rear wheels is 11 feet, and the distance between the central planes of the rear wheels is 5 feet. Johnson, in "Modern Framed Structures," p. 44, gives a diagram from which values of $w$ can be read off more accurately than by formula (2), not only for case A, to which the formula applies, but also for cases B and C, which include county bridges.

The last-named authority is quoted below, for the weights of railroad bridges corresponding to Cooper's class, "Extra Heavy A" loading. The formulas below, for dead load per *linear foot*, are for two trusses only and single-track bridges; for double-track, add 90 per cent.

For deck-plate girders:

$$w = 9\,l + 520 \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

For lattice girders:

$$w = 7\,l + 600 \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

For through pin-connected bridges:

$$w = 5\,l + 750 \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

For Howe trusses of wood:

$$w = 6.2\,l + 675 \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

In all these formulas, the track has been included, being estimated at 400 pounds per linear foot, as is usual in computing stresses. Its average value is nearer 300 pounds.

*Example I.*—What is the weight of a through pin-connected railroad bridge entire (track included) of 100 ft. span?

From (5) $w = 5 \times 100 + 750 = 1250$ pounds per linear foot; weight of bridge is $1250 \times 100 =$ 125,000 pounds, half of which, or 62,500 lbs., is carried by one truss.

*Example II.*—What is the weight per foot of a highway bridge of 100 ft. span, roadway 18 ft. wide? In (2) put $b = 18$, $l = 100$ and get $w = 676$ ∴, weight on one truss $= 676 \times 100 \div 2 = 33{,}800$ pounds.

6. The weight acting on one truss is carried to the apices of upper and lower

chords. Thus, in fig. 1, half the weight of web members, the weight of upper chord and half the upper transverse bracing will be supposed borne at the upper chord apices. Similarly the weights of half the web members, lower chord and lower transverse bracing, will be borne at lower chord, and in addition a panel length of the half roadway. Thus at an apex *a*, if *b* and *c* are at the middle of adjoining panels, the weight of roadway on a panel length, *b a c*, is carried first to the floor beam, and then half this weight is transferred by the floor beam to apex *a*, and the other half to the apex of the truss opposite.

The chord that carries the roadway is thus the most heavily loaded; and it is usual to consider that it carries, at its apices, two-thirds of the dead load per panel, and the other chord apices one-third of the same.

If we suppose fig. 1 to represent a highway bridge of 100 span, 18 feet between trusses, we have just found (see Ex. II. above) the weight per linear foot of bridge on one truss to be 338 pounds. As there are 6 panels, the weight per panel is 338

$\times 100 \div 6 = 5633$ pounds. Two-thirds of this, or 3755 pounds, is supposed, to be carried at each lower apex, and one-third, or 1878 pounds, at each apex of the top chord, except at the ends, where only half or 939 pounds is allowed, as the end apices of top chord carry only half the weight of top chord and transverse bracing borne at the other apices. This is only roughly approximate, as these end apices really carry as much of the web as the interior apices; still the results are near enough for computation.

If we call the dead loads sustained at the lower apices $w_1$, $w_2$, $w_3$, $w_4$ and $w_5$, each will equal 3755 pounds. If we call the loads sustained at the apices of top chord $w_6$, $w_7$, $w_8$, $w_9$, $w_{10}$ and $w_{11}$, then $w_6 = w_{11} = 939$, and $w_7 = w_8 = w_9 = w_{10} = 1878$ pounds. The half-panel loads in end lower panels are borne directly to the abutments at A and B, and are not considered in the computation of stresses in the truss members. Thus the load on this truss will be taken as $5 \times 3755 + 4 \times 1878 + 2 \times 939$, or $5 \times 5633 = 28{,}165$ pounds. The reaction at each abutment is one-half this, or 14,082

pounds, since the loads are symmetrically distributed as to the centre of the truss.

LAWS OF MECHANICS AND APPLICATIONS.

7. In addition to the principle of the triangle of forces, the following laws of mechanics will be used:

If any number of forces, acting on a rigid body and in the same plane, are in equilibrium,—

LAW I. *The algebraic sum of their moments about any point in the plane of the forces is zero.*

LAW II. *The algebraic sum of their vertical components is zero.*

LAW III. *The algebraic sum of their horizontal components is zero.*

Calling, as above, $w_1, w_2, \ldots$, the dead panel loads on a truss, fig. 1, and $x_1, x_2, \ldots$, the distances to the right abutment (centre of end pin); also calling $w', w'' \ldots$, the part of any live load borne by one truss and $x', x'', \ldots$, the distances to right abutment from their centres of gravity respectively, we have by Law I., calling R

the left vertical reaction, $R_1$ the reaction at B, and $l$ the span, and taking B as a centre of moments:

$$R \times l - (w_1 x_1 + w_2 x_2 + \ldots) - (w' x' + w'' x'' + \ldots) + R_1 \times o = o$$

Whence,

$$R = \frac{(w_1 x_1 + w_2 x_2 + \ldots) + w' x' + w'' x'' + \ldots}{l} \quad \ldots(7)$$

Or, in words: to find the left vertical reaction, add the products obtained by multiplying each weight or load by its horizontal distance to the right abutment, and divide the sum by the span.

As the reaction due to dead load on one truss, when it is symmetrical as to the centre, is always one-half the sum of the dead panel loads, it is generally computed separately and added into the reaction found by the above rule for the live load.

In railroad bridges, $w'$ $w''$ ..., are actual loads on successive wheels, until the specified uniform car load is reached, when the car-load on the bridge is treated as a single force acting through *its centre of gravity*. This subject will be developed

more fully later on. In highway bridges it is customary to compute live panel loads for the load per square foot allowed on the clear width of floor between trusses, and to regard these loads per panel concentrated at the apices of the loaded chord. This may be called the method of apex loads, which last are supposed to move a panel length at a time. The distances $x'$, $x''$ ..., as well as $l$, should then be given in panel lengths, regarding a panel length as the unit of length.

*Ex.* When the above formula or rule is applied to the dead load only of the truss, why does it give R = half dead load ?

8. If we call W the total load, live and dead, on the truss, we have by Law II.

$$R + R_1 - W = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Therefore, when R is known from (7), $R_1$ can be found.

9. The truss, fig. 1, will now be supposed to be severed along the line *d e*. Suppose the part of the truss to the right of *d e* removed, and that forces C, D, T, are applied at the cut parts, equal and

directly opposed to the resistance of those members (fig. 3). These forces will act in

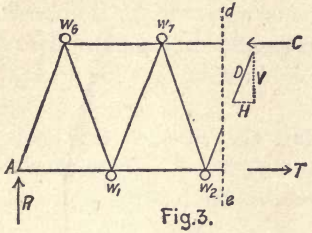

Fig.3.

the direction of the cut members, and will exactly represent the action of the right part of the truss upon the left at the section.

The forces C, D, T, the reaction R and the loads (live and dead) on the part of the truss left of the section *d e*, thus constitute a system of forces in equilibrium acting on the structure A *d e*.

Denote the vertical component of D by V, its horizontal component by H; then since the upper chord is in compression, the lower in tension, C must act to the left and T to the right, and we have, for horizontal chords, by Law III., if D is sup-

posed to act downwards, and ∴ H to the left,

$$-C-H+T=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

If D acts upwards, H is directed to the right and is plus in eq. (9). The direction in which D acts is shown in the next article.

10. Call the sum of the loads on the left of the section *d e*, $\Sigma w$; then the difference between the left reaction and $\Sigma w$, or

$$R-\Sigma w,$$

is called the *shear* left of the section Law II. applied here may be stated as follows: for equilibrium, if the shear $(R-\Sigma w)$ is plus, it indicates that it acts *up*, hence V must act *down*; therefore D acts down, and the diagonal cut by the section *d e*, is in *compression*; but if the section is taken far enough to the right, so that $(R-\Sigma w)$ is minus, the shear just to left of section acts *down*, therefore V must, for equilibrium, act *up*; hence D acts up and the diagonal is in *tension*.

The above laws hold when the top of the diagonal leans *away from* the *left abutment*, as in fig. 3.

11. Let the section be now passed cutting a diagonal whose top leans *towards* the *left abutment*, as in fig. 4.

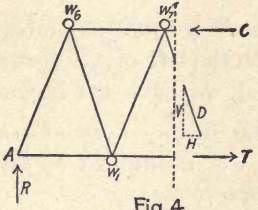

Fig. 4

Here, if the shear just left of the section $(R - \Sigma w)$ is plus, it acts *up*, $\therefore$ V and therefore D, acts down; hence, the diagonal cut is in tension; but if $(R - \Sigma w)$ is minus, the shear acts *down*, and hence V and D act *up*, and the diagonal cut is in compression.

These most important rules should be thoroughly understood by the reader, who should be capable of immediately applying them to find the amount and character of the stress in any web member. The rules apply equally in case any of the web members are vertical. In this case, V = D = stress in vertical member.

12. If $i$ is the inclination of any diagonal to the vertical, the stress in it is,

$$D = V \sec. i;$$

hence, since V is equal numerically to the shear just to the left of the section cutting the diagonal, we have the important rule:

*The stress in any diagonal is equal to the shear on it multiplied by the secant of its inclination to the vertical.*

*When the top of the diagonal leans towards the left abutment, and the shear from the left is plus, it is in tension; if the shear is minus, it is in compression. But if the top of the diagonal leans from the left abutment and the shear is plus, the diagonal is in compression; if the shear is minus, it is in tension.*

*Example I.*—Draw fig. (1) to a larger scale and write the panel dead loads given in Art. 6 at each apex, then number the diagonals from left to right, 1, 2, 3 . . ., and find the shears and character of the stresses in each diagonal.

In the table below, the shear in (1) is the left reaction, 14,082. From this we subtract 939 (load at first upper apex) to find shear in (2). From

shear in (2) we subtract load 3755 at first lower apex to find shear = (R — $\Sigma w$) in (3), and so on:

| Diagonal. | Shear. | Diagonal. | Shear. |
|---|---|---|---|
| 1 | + 14,082 | 12 | — 14,083 |
| 2 | + 13,143 | 11 | — 13,144 |
| 3 | + 9,388 | 10 | — 9,389 |
| 4 | + 7,510 | 9 | — 7,511 |
| 5 | + 3,755 | 8 | — 3,756 |
| 6 | + 1,877 | 7 | — 1.878 |

As we subtract apex loads in turn from the preceding shear, we find the shear first minus in (7). From shear in (7) subtract 1878 to get shear in (8); from this 3755 to find shear in (9), and so on for minus shears. The character of the stresses is now given by the rule above, viz.: (1) and (12) compression, (2) and (11) tension. That is, the members are alternately in tension and compression.

The reaction or shear in (1) is strictly 14,082½. If this correct value is taken, the shears in (1) and (12), (2) and (11), or members equally distant from the centre, are all equal. Hence, the stresses in them are equal; and we have seen, too, that they are of the same character.

This must likewise follow, because the truss and loads are symmetrical about the centre.

*Example II.*—Suppose sections passed through diagonals (4) and (9), construct the two figures to

left of section and supply forces as in Arts. 10 and 11 (figs. 3 and 4), find value and character of stresses in diagonals (4) and (9) when *sec.* $i = 1.25$. Do the same for diagonals (5) and (8).

## Position of Live Load, Giving Maximum Positive Shear in a Web Member.

13. The case of actual wheel loads will be taken up later. In this article the uniform live load per panel will be regarded as concentrated at the apices of the chord which bears the roadway. Further on the actual position of the moving load will be considered.

*The shear for equal live apex loads is a positive maximum when the apex loads extend from the farthest right abutment to the section cutting the web member, with none of the loads lying on the left of the section.*

Referring again to fig. 1, the largest positive shear occurs in the diagonal cut by the section $d\,e$, when the three lower apices to the right of the section are loaded, each with a panel live load.

(*a*) For as the shear in the cut diagonal

is R — $\Sigma w$ (Art. 10); if any apex load on the part B $d e$ is taken off, R is diminished, but $\Sigma w$ (the sum of the loads left of the section) remains the same; hence the shear is less than when such apex to the right of the section is loaded.

(*b*) Again, if any apex live load is added to the left of the section $d e$, R is increased by a part of it only, whereas $\Sigma w$ is augmented by the whole of it, so that the shear R — $\Sigma w$ is less than before. We conclude as enunciated.

If we suppose the truss temporarily without weight with live apex loads on the left of the section, thus covering the shorter segment and none on the right, the shear is negative, as seen in case (*b*), and a maximum. As the shear along $d e$, due to dead load, is plus (as $d e$ is to the left of the centre) the two shears conspire against each other, whereas when the longest segment alone is loaded, the shear due to both live and dead loads is positive and their sum is the total shear We shall see later on the significance of these remarks.

### Warren Truss with Suspenders.

14. Figure 5 represents a triangular truss with parallel chords of 180 ft. span divided into 12 panels of 15 feet each. The

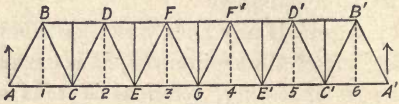

Fig. 5.

height of truss, fig. 5, centre to centre of chords, is assumed at 26 feet, giving the length of a diagonal 30 feet and *sec.* $i = 30 \div 26 = 1.154$.

If this is regarded as a highway bridge, 20 ft. between trusses in the clear, the dead load per linear foot on one truss is given by (2) Art. 5, as,

$$70 + 6\ \mathrm{b} + 0.1\ b\ l - 0.2\ l,$$

or for $b = 20$, $l = 180$, 514 pounds per foot. The panel load is thus, $514 \times 15 = 7710$ lbs., of which $\frac{2}{3}$ or 5140 lbs., goes to each lower apex and $\frac{1}{3}$ or 2570, to each upper apex.

The live load will be assumed at $93\frac{1}{3}$ pounds per square foot of roadway, which

gives $\frac{1}{2}(15\times 20+93\frac{1}{3})=14{,}000$ pounds for the panel or apex live load, which acts only at the apices of the lower chord.

The vertical suspenders at B, D, F..... simply hold up 5140 lbs. dead and 14,000 lbs. live load when fully loaded.

The vertical posts at C. E....., are only intended to support the upper chord, and hence sustain 2570 lbs., though it is really less than that, as the upper lateral bracing will be supposed connected to the upper chord only at points B, D, F....., and hence the vertical posts do not get the full allowance of $\frac{1}{2}$ the dead panel load. It remains now to compute the stresses in the diagonals and chords.

15. *Diagonals.* The left reaction due to dead load acting at 11 lower apices and 11 upper, is $7710\times 11\div 2=42{,}405$, and this is the dead load shear in A B. By subtracting the sum of the loads at apex B and the apex vertically under it, or 7710 lbs., we get the shear in B C. Continuing thus to subtract 7710 from each shear in turn, we find all the dead load shears given in the table below.

The reaction due to the live load when it covers 11 lower apices, is $14{,}000 \times 11 \div 2 = 77{,}000$ lbs., the live load shear in A B. It is most convenient in what follows to use a panel length as the unit of length.

For the maximum shear in B C, by Art. 13, the live load must extend from the farthest abutment to C. The reaction, which is the shear for live load only, is less than before by the amount of reaction due to the apex load to left of C, which has been taken off, or $\frac{11}{12}(14{,}000) = 12{,}833$, giving the reaction 64,167 lbs., which is the maximum shear due to live load only in B C.

For C D we similarly take off load at C, whose reaction $\frac{10}{12}(14{,}000) = 11{,}667$ must be subtracted from preceding shear 64,167 to get the shear 52,500 lbs. in C D.

Continuing thus, as the load backs off to the right, we subtract in order,

$$\frac{9}{12}, \frac{8}{12}, \frac{7}{12}, \frac{6}{12}, \ldots, \frac{1}{12}$$

of 14,000, and thus get the live load shears in the table.

| Front of live load at | Member. | Dead load shear, lbs. | Live load shear, lbs. | Total shear, lbs. | Character of stress, $c$ = compression, $t$ = tension. |
|---|---|---|---|---|---|
| 1 | A B | +42,405 | +77,000 | +119,405 | $c$ |
| C | B C | +34.695 | +64,167 | + 98,862 | $t$ |
| 2 | C D | +26,985 | +52,500 | + 79,485 | $c$ |
| E | D E | +19,275 | +42,000 | + 61,275 | $t$ |
| 3 | E F | +11,565 | +32,667 | + 44,232 | $c$ |
| G | F G | + 3,855 | +24,500 | + 28,355 | $t$ |
| 4 | F′ G | − 3,855 | +17,500 | + 13,645 | $c$ |
| E′ | E′ F′ | −11,565 | +11,667 | + 102 | $t$ |
| 5 | D′ E′ | −19,275 | + 7,000 | − 12,275 | $t$ |
| C′ | C′ D′ | −26,985 | + 3,500 | − 23,485 | $c$ |
| 6 | B′ C′ | −34,695 | + 1,167 | − 33,528 | $t$ |
| A′ | A′ B′ | −42,405 | + 0 | − 42,405 | $c$ |

The total shears are found by adding algebraically those due to dead and live loads respectively. The character of the stress is given by the rule in Art. 12. The live load shears being formed here, by subtracting successively $\frac{11}{12}$ (14,000), $\frac{10}{12}$ (14,000),.....from the preceding shear,

when the result for $A^1$ $B^1$ comes out zero, it shows that the work is correct, as no live load is then on the bridge.

These subtractive terms can easily be found by adding $\frac{1}{12}$ (14,000) = 1167 to itself to get $\frac{2}{12}$ (14,000), then 1167 to this to get $\frac{3}{12}$ (14,000), and so on.

Throwing away decimals will always cause such results to differ a few pounds from the true ones, which is of no consequence in practice.

If desired, any live load shear can be found independently. Thus, to get the shear in F′ G due to loads of 14,000 lbs. each at 4, E′, 5, C′, 6; note that the resultant of the five loads acts at 5. Hence the left reaction is $\frac{8}{12}$ of (5 × 14,000) = 17,500, and this is the live load shear in F′ G for this position of the load.

The total shears given in the table for the diagonals left of the centre are maximum shears, as the live load extends from the farthest (right) abutment to the diagonal considered.

16. *Counter-Braces.*—By the rule of Art. 12, when the live load extends from

A′ to 4, since the shear in F′ G is + 13,645, this piece is in compression and subjected to a stress of 13,645 *sec. i* = 15,746 pounds.

Similarly, when the live load extends from A′ to E′, the diagonal E′ F′ is under a tension of 102 *sec. i* = 118 pounds.

If any of the live load was taken off to the right of the sections cutting these members, or any added to the left of the sections, the live load + shear is less (Art. 13); and hence, since the dead load shear is minus, the total shear if positive would be less. The maximum compression that F′ G can ever experience is thus 13,645 *sec. i* pounds, and the maximum tension that E′ F′ can ever sustain is 102 *sec. i* pounds. Now, since F′ G and F G are symmetrically situated with respect to the centre G, when the live load extends from A to 3, F G must sustain a compression of 13,645 *sec. i*, the same that F′ G sustains when the live load extends from A′ to 4.

The piece F G must then be designed to sustain not only the maximum tension of 28,355 *sec. i* pounds, given in the table,

when the live load extends from A′ to G, but also a maximum compression of 13,645 *sec. i* pounds when the live load extends from A to 3.

Similarly, the diagonal E F must not only be designed to sustain the compression given in the table, of 44,232 *sec. i* pounds, when the live load extends from A′ to 3, but likewise a tension of 102 *sec. i* when the live load extends from A to E, for the last stress must equal that in E′ F′, in amount and character, for a live load extending from A′ to E′, since the members E F and E′ F′ are symmetrically situated as to the centre G. The diagonals E F, E′ F′, F G, F′ G′ are thus "counter-braces" (Art. 3), and we shall find that these are the only members in the truss requiring counter-bracing.

17. *Minimum Stress in a Web Member.*—Recurring to the table, we see that diagonals D′ E′, C′ D′, B′ C′ and A′ B′, when the live load extends from A′ to 5, C′, 6 and A′, respectively, receive the same character of stress as when subjected to dead load only, which likewise gives a

minus shear for each of them, as given in the table. These positions of the live load (extending to the nearest abutment) cause the *minimum* stress the members in question ever receive, for the + shear is greatest for the positions given, since any live load taken off the right of the section or added to the left diminishes the + shear (Art. 13), and the greater the + shear the less the "total"—shear, as we see from the table.

Now, from considerations of symmetry, diagonal B C with live load from A to 1, receives the same tension as B′ C′ with live load from A′ to 6. Similarly, C D receives the same compression with live load from A to C, as C′ D′, when the live load extends from A′ to C′, and in the same way, minimum stress in D E = stress in D′ E′, as found from the table.

We can now make out the table for maximum and minimum stresses in the web members, together with the extremes of stress in the counter-braces, by multiplying the shears in the table above by *sec.* $i = 1.154$.

| | A B | B C | C D | D E |
|---|---|---|---|---|
| Max... | — 137,793 | + 114,087 | — 91,726 | + 70,711 |
| Min... | — 48,935 | + 38,691 | — 27,102 | + 14,165 |

COUNTER-BRACES.

| | | E F | F G |
|---|---|---|---|
| Max... | ..... | — 51.043 | + 32 722 |
| Min... | ..... | + 118 | 15,746 |

The plus sign is here used to indicate tension; the minus sign, compression.

18. If there should be any difficulty in comprehending the subject of counter-braces and minimum stress, it may be entirely removed if the reader will make a rough sketch of the truss, large enough to mark alongside each diagonal the amount of the total shears (without sign) given in the first table above and the character of the stress, *t* for tension, *c* for compression.

These shears correspond to the live load extending from the right abutment to the diagonal in question.

Then suppose the live load to extend from the left abutment to the diagonal considered and again mark the shears

(without sign) on each diagonal and the character of the stress as given in Arts. 16 and 17. Or, if preferred, we can suppose the truss turned end for end, so that A′ is at A and A at A′. Then as the live load backs off to the right, the same total shears, on the diagonals at the left end of the live load, will be experienced as before, for members equally distant from the left abutment. Mark these total shears (without sign) and the character of the stress from the first tracing above, on a second drawing placed below the first. Thus the maximum shear in C′ D′ of the second drawing is now 79,485, the same previously marked on C D the member just above it on the first drawing and the piece is in compression as previously marked on C D. Turn the second drawing end for end and then write the shears and character of the stress, as taken from it, on the members just above on the first drawing. These last shears thus correspond to the live load on the original truss backing off the span to the left.

It will now be seen that pieces A B, B

C, C D, D E, and A′ B′, B′ C′, C′ D′, D′ E′ are subjected to but one kind of stress, whereas the remaining diagonals are subjected in one case to tension, and in the other to compression, and hence must be designed as counter-braces. The first set of shears are experienced as the live load, moving from right to left, reaches each diagonal in turn; the second set as the live load, moving from left to right, reaches each diagonal in turn. The stresses are those given in the second table above.

This subject has been carefully given, and should be thoroughly understood before taking up other trusses where a knowledge of it is assumed.

### Chord Stresses by the Method of Moments.

19. To prove that each apex of the chord that bears the roadway must be loaded to cause maximum stresses in the chords, consider the truss without weight, and that a single apex load $w$ rests anywhere to the right of the centre of moments E for finding the stress in D F.

Pass a section cutting D E, C E and D F and supply forces as in Fig. 4, Art. 11. The reaction R gives a right-handed moment R × A E about E, which must be resisted by a left-handed moment, hence the force applied to the cut chord D F acts to the left, giving compression in D F and its moment is (the stress in D F) × $h$, ∴ compression in D F = R × A E ÷ $h$. As R increases with every load added to the right of E, so does the compression in D F.

Next consider apex loads to left of E, from E to A′ being unloaded. The part of the truss left of the section may now be removed and forces applied to cut parts equal to the forces caused by the left part on the cut members.

The moment of forces to right of section about E is simply $R_1$ × A′ E, where $R_1$ is the right reaction. This left-handed moment must be resisted by a right-handed moment equal to force applied to chord D F at the section, multiplied by $h$. The last force must, therefore, act to right, which causes a compression in chord D F = $R_1$ ×

A′ E ÷ $h$. As this increases with $R_1$ the segment A E must be fully loaded for a maximum compression in D F. Therefore, we conclude that any load on the bridge (live or dead) causes a compression in D F, and that the latter is a maximum when the bridge is fully loaded.

Similarly, it can be shown that any load on the bridge causes tension in the lower chord, which is a maximum for the bridge fully loaded.

20. To find the chord stress in any panel of upper or lower chord, due to the dead load and a full live load, pass a section cutting only three pieces; the chord piece in question, a diagonal and the other chord, as in figs. 3 or 4 (arts. 9 and 11), and suppose the right part of the truss removed. Then supplying forces equivalent to the forces the right part exerts upon the cut members, we have a system of forces in equilibrium acting upon the part of the truss left of the section.

The simple rule now is: To find the stress in either chord piece cut by the section, take as a centre of moments the in-

tersection of the web member severed and the other chord, and then express that the moment of the stress in the chord equals the algebraic sum of the moments of the other forces acting on the part of the truss considered. The upper chord will be assumed in compression, the lower in tension, as found above.

The forces acting on the other chord-piece and diagonal pass through the centre of moments, hence their moments are zero; therefore, we have simply to find the moments due to the reaction and loads left of the section and put their algebraic sum equal to the moment of the chord stress. The live and dead loads on the same vertical line have the same arm, hence we shall call P the sum of the live and dead panel loads at any lower apex and the apex vertically over it, $\therefore P = 14{,}000 + 7710 = 21{,}710$ pounds. The panel length $d = 15$ feet and the height of truss, centre to centre of chords, is $h = 26$ feet. The reaction due to 11 loads of 21,710 lbs. each is $R = 119{,}405$ pounds.

*Upper Chord Stresses.*—B D. Pass section

cutting B D, either B C or C D and the lower chord and take C for centre of moments; then, stress in B D $\times h = R \times 2 d - P \times d$.

D F. centre moments at E, stress in D F $\times h = R \times 4 d - 3 P \times 2 d$, since the resultant of P at 1, C and 2, is 3 P acting at C, 2 $d$ to left of E.

$F F'$. Centre of moments at G, stress in F F$'$ $\times h = R \times 6 d - 5 P \times 3 d$, since P at 1, C, 2, E and 3, gives a resultan 5 P at 2, on account of symmetry of loads about 2.

*For the lower chord:*

A C. Moments about B.

Stress in A C $\times h = R \times d$.

C E. Moments about D.

Stress in C E $\times h = R \times 3 d - P (1 + 2) d$.

E G. Moments about F.

Stress in E G $\times h = R \times 5 d - P (1 + 2 + 3 + 4) d$.

Substitute $R = 5.5 P$ in all the above equations and simplify.

On substituting numerical values we find the stresses in the chord members as follows, the upper chord being in compression, the lower in tension:

| | Stress. | $\triangle_1$ | $\triangle_2$ |
|---|---|---|---|
| A C | 68,893 | | |
| B D | 125,260 | 56,367 | 12,526 |
| C E | 169,101 | 43,841 | 12,526 |
| D F | 200,416 | 31,315 | 12,526 |
| E G | 219,205 | 18,789 | 12,526 |
| F F′ | 225,468 | 6,236 | |

On finding the differences, $\triangle_1$, between successive chord stresses, and taking the difference of these successive differences, we note that they are constant, which proves the numerical work, as will be shown later on (Art. 23).

### Chord Stresses by the Increment Method.

21. Another method for finding chord stresses, applicable only to horizontal chords, will now be given.

For full live and dead load, call, as before, the sum of the live and dead apex loads at any lower apex and the apex dead load at the upper apex vertically over it = P = 21,710 pounds, and find the shears in the diagonals by subtraction. Thus shear in,

| | |
|---|---|
| A B = 5.5 P | D E = 2.5 P |
| B C = 4.5 P | E F = 1.5 P |
| C D = 3.5 P | F G = 0.5 P |

Now pass a (curved) section cutting members meeting at B (fig. 6), supply

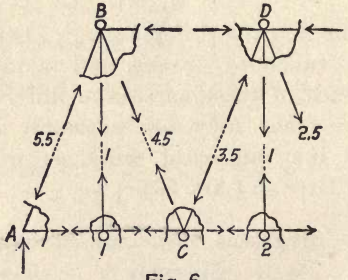

Fig. 6.

forces at the cut sections equal to the forces caused by the balance of the truss

on these members, and let the balance of the truss be removed. We have thus a force acting up along A B, whose vertical component is 5.5 P; a vertical force acting down along B 1; a force acting down along B C, whose vertical component is 4.5 P, and a force acting to left along B D. The directions of the forces follow from the fact that A B and B D are in compression, B C and B 1 in tension (Art. 12).

Since we have a system in equilibrium, the sum of the horizontal components is zero; but the horizontal component of any diagonal whose vertical component is V, is V *tan i*, where *i* is the angle it makes with the vertical, ∴ stress in B D = (5.5 P + 4.5 P) *tan i* = 10 P *tan i*. Similarly, pass a curved section cutting all members assembling at D, supply forces, etc., as before. The force equal to the compression in the chord D F acts to the left; that acting on D B acts to right and is equal to the compression in B D = 10 P *tan i*, just found. The horizontal components of

forces applied to diagonals, both act to right, since C D is in compression and D E in tension (Art. 12). Hence by law III. (Art. 7), stress in D F =

$$(10 + 3.5 + 2.5)\ \mathrm{P}\ tan\ i = 16\ \mathrm{P}\ tan\ i.$$

We proceed similarly at joint F, giving stress in F F′ =

$$(16 + 1.5 + 0.5)\ \mathrm{P}\ tan\ i = 18\ \mathrm{P}\ tan\ i.$$

*The chord stress is here a maximum where the shear is zero.*

A similar method applies at apices A, C and E, of the lower chord.

Note that at A, the force supplied along B A acts down (fig. 6) ∴ tension in A C = 5.5 P *tan i*.

At joint C (fig. 6), the force supplied along D C acts down, that along B C up, and the one (5.5 P *tan i*) just found along A C, to the left; since D C is in compression and both B C and A C are in tension. The horizontal components of all these forces therefore act to the left ∴ tension in C E =

$$(5.5 + 4.5 + 3.5)\ \mathrm{P}\ tan\ i = 13.5\ \mathrm{P}\ tan\ i.$$

We have similarly, at apex E, tension in E G =

$$(13.5 + 2.5 + 1.5)\ P\ \tan i = 17.5\ P\ \tan i.$$

We have $P\ \tan i = 21\,710 \times \frac{15}{26} = 12{,}525$, hence the stresses are as follows:

| | | | |
|---|---|---|---|
| B D | = 125,250, | A C | = 68,887, |
| D F | = 200,400, | C E | = 169,087, |
| F F′ | = 225,450, | E G | = 219,187. |

The slight differences between these values and those found in the preceding article are due to neglecting decimals in the provious case.

The chord stresses in members to the right of the centre, under full load, are of course the same as for members the same distances from the centre and to its left.

22. The above method, by chord increments, is general for all trusses with horizontal chords and may be stated as follows:

*To find the stress in any chord panel, compute the shears in all the web members to its left, including those meeting at the left end of the chord panel, and multiply each shear by the tangent of its angle with*

*the vertical. The sum of the products is the stress in the chord.*

It is usual to test the stress thus found in the central panels by the principle of moments given above, and this should never be omitted, as it affords a check on the entire work.

23. Another method of checking a part of the work can be derived as follows:

Write down the coefficients of $P \tan i$ for the chord panels, first a lower panel, then the next upper, and so on.

Find the first and second differences, and note that the last are all equal.

| | Coeff's of P *tan i.* | $\triangle_1$ | $\triangle_2$ |
|---|---|---|---|
| A C, | 5.5 | | |
| B D, | 10.0 | 4.5 | 1 |
| C E, | 13.5 | 3.5 | 1 |
| D F, | 16.0 | 2.5 | 1 |
| E G, | 17.5 | 1.5 | 1 |
| F F, | 18. | 0.5 | |

The constant second differences are thus P *tan i* = 12,525, as found in Art. 20.

24. The *minimum* stress in the chords is evidently that due to dead load alone, and the ratio of minimum to maximum stress is simply the ratio of a panel of dead load to a panel of dead and live load. Thus this ratio, in this case, is 7710 ÷ 21,710 = 0. 355.

This ratio is useful in fixing the unit stress to allow in the chords, as the unit stress should be made less the greater the extremes of stress.

### Formulas for Chord Stresses.

25. Figures 7 and 8 represent two trusses with horizontal chords. The first is called a Pratt truss and the second a Howe truss. The first is supposed a truss of a deck bridge, the latter of a through bridge; but we shall see that the formulas deduced for chord stresses below apply equally if the first belongs to a through bridge and the last to a deck bridge.

Let the trusses be loaded with their own weight, supposed concentrated at upper

and lower apices according to a suitable convention, and also with a uniform live

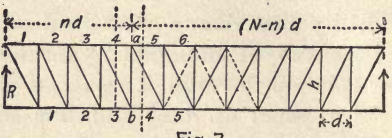

Fig. 7.

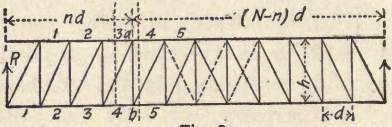

Fig. 8.

load concentrated at the apices of the chord that bears the roadway.

Call P = total dead and live load concentrated at any apex and the one just above it.

N = total number of panels (12 in the figures).

$n$ = number of panels from left abutment to centre of moments.

$h$ = height of truss, centre to centre of chords.

$d$ = panel length.

$t_n$ = stress on $n^{th}$ panel of lower chord,
$c_n$ = stress in $n^{th}$ panel of upper chord.

The dotted diagonals in the four central panels are called *counters*, and are not in action under a uniform load; for an investigation similar to that in Art. 12, Ex. I. will show that all the full line diagonals in fig. 7 are in tension, and all the full line diagonals in fig. 8 in compression. The full line diagonals then in fig. 7 will stretch, hence the dotted diagonals (counters) will be shortened by the distortion of the panel, and as they are always of very small section, being designed as ties only, they can take no appreciable compression, and hence may be considered out of action. The counters in fig. 8 are simply struts, abutting at their ends against angle blocks, and not fastened to them in any way; consequently when the main diagonals are shortened by the compression, the distortion of the panel lengthens the distance between the apices at the ends of the counters, so that the latter are no longer in action. Leaving out the counters, therefore, we conceive

the truss cut in two, along the dotted lines, through the panel marked 4 of the upper chord in both figures, and the right part of the trusses removed. Then apply forces equal and opposite to the resistance of the cut members, and take the centre of moments, for the forces acting on the left part at *b*, the intersection of the diagonal and lower chord to find the compression $c_n$ in the 4th panel of the upper chord.

As the force $c_n$ applied to the cut top chord acts to the left (the top chord being always in compression, Art. 19), its moment $c_n h$, is left handed, and it must equal the right-handed moment of the left reaction and loads to the left of *b*. Noting that the distance A *a* is $n d$, the moment of the left reaction R about *b*, is R $n d$, $n$ in this case being 4. The sum of the loads between A and *a b* is $(n-1)$ P, and it is seen that their resultant acts at $\frac{1}{2}$ $n d$ from *a b*, hence the moment of their resultant about *b* is $(n-1)$ P $\frac{1}{2}$ $n d$

Therefore, since $R = (N-1)\frac{P}{2}$ we have

$$c_n h = R\, n\, d - (n-1)\, P\, \tfrac{1}{2}\, n\, d$$

$$= [(N - 1)\, n - (n - 1)\, n] \tfrac{1}{2} P d$$

$$\therefore c_n = (N - n)\, n \frac{P d}{2 h} \ldots\ldots (10)$$

This formula was deduced for $n = 4$, but it is equally true for any value of $n$; for if we wish to find $c_5$, for example, the centre of moments is horizontally $n\,d\,(=5\,d)$ from the left reaction, so that its moment is $n\,d\,R$. There are again $(n - 1)$ loads P to left of centre of moments, and they are symmetrically disposed about a point midway between A and the centre of moments, so that the arm of the resultant is $\frac{1}{2}\,n\,d$ and the sum of their moments $(n - 1)\,P\,\frac{1}{2}\,n\,d$, as before.

It is always understood, in writing down general formulas of this kind, that the reader will test their generality by imagining $n$ to increase or decrease by 1, and seeing that the results are true, no matter what $n$ may be.

To find the stress $t_n$, in the $n^{th}$ panel of the lower chord, we suppose a section taken across the $n^{th}$ panel of lower chord, say the one marked 4 in the figures. The

centre of moments is now at $a$, and the forces and their arms are given by the same formulas as above; therefore we deduce as before,

$$t_n = (N - n)\, n \frac{Pd}{2h} \ldots\ldots (11)$$

Therefore, to find the chord stresses in panels marked 1, 2, ....., of the upper chord, or of those marked on the figures, 1. 2, ....., of the lower chord, we substitute successively $n = 1$, $n = 2$, etc., in formula (10) or (11). The upper and lower chord panels similarly numbered on the two figures thus have the same stresses, a result which follows from Art. 9, eq. 9, likewise, if we pass a section cutting any two chord pieces, marked with the same number, and the vertical member between them and place the sum of the horizontal components equal to zero

$$t_n - c_n = o, \therefore t_n = c_n \ldots$$

*Example.*—As in Art. 20, take $N = 12$, $h = 26$, $d = 15$, $P = 21{,}710$ pounds. On substituting numerical values in (10) and (11), we find the same chord stresses given in Article 20. In fact, the formulas can easily be proved to apply to Fig.

5, Art. 20, if we number the upper chord panels B D, D F, F F', 2, 4, 6 and the lower A C, C E, E G, 1, 3 and 5, and thus find for the upper chord $c_2$, $c_4$, $c_6$ from (10) and for the lower chord $t_1$, $t_3$, $t_5$ from (11). This follows, as will be seen in going over the proof of the last article, because $n\,d$ will always give the arm of the reaction for the chord panel marked $n$, and the $(n - 1)$ loads P are similarly situated in all three trusses.

Prove the formulas (10) and (11) applicable to fig. 5 (Art. 20) in the manner stated.

26. *From the method of derivation of eqs.* (10) *and* (11), *it is seen that they always give the stress in a chord member whose centre of moments is* n d *from the left abutment, the loads being placed as in figures* 7 *and* 8.

The formulas (10) and (11) can be put under another form if we let $m = \mathrm{N} - n$ = number of panels to right abutment from the centre of moments, and remember that $n$ = number of panels from left abutment to centre of moments, for the chord-piece considered.

$$t_n = c_n = n\,m\,\frac{\mathrm{P}\,d}{2\,h} \quad \ldots\ldots (12)$$

*Example 1.*—In the Warren girder, Fig. 1 of

Art. 2, of 6 panels, let $d = h = 10$ and let P = 2 tons, represent the panel live load at each apex of the lower chord, the dead load not being considered.

We can find by (12) above the stresses in the top chord, but not in the lower. Why?

For first upper chord panel, the centre of moments is one panel length from A or $n = 1$, $\therefore c_1 = 1 \times 5 = 5$ tons.

For second panel, centre is 2 $d$ from A or $n = 2$, $\therefore c_2 = 2 \times 4 = 8$ tons.

For third panel, $c_3 = 3 \times 3 = 9$ tons.

Find by the method of moments of Art. 20 these values, also the lower chord stresses, $t_1 = 2.5$, $t_2 = 6.5$, $t_3 = 8.5$.

*Example II.*—Find by the method of chord increments (Art. 22) the same chord stresses in last example.

## 27. Second Difference Constant.

After computing the chord stresses in panels 1, 2, 3, . . . , in turn (fig. 7 and 8), it is well to check by taking first and second differences, as in Art. 20. The second differences are constant and equal to $P \frac{d}{h}$

Thus:

$$c_n = (N - n)\, n \frac{Pd}{2h}$$

$$c_{n+1} = (N - n - 1)\ (n+1)\ \frac{P\,d}{2\,h}$$

$$c_{n+1} - c_n = (N - 2\,n - 1)\ \frac{P\,d}{2\,h}$$

This first difference changes (2) $\frac{P\,d}{2\,h} = P\,\frac{d}{h}$ for each increase of $n$ by 1, $\therefore$ the second difference is constant.

In the example given in Art. 25 the second difference $P\,\frac{d}{h} = 12{,}525$ as before found.

### Warren Truss.

28. The chord stresses in the Warren Deck Truss, fig. 9, having diagonals

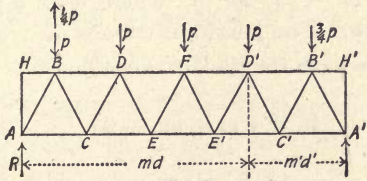


Fig. 9.

equally inclined to the vertical, are easily made out by the method of chord incre-

ments, Art. 22, but we shall deduce a formula for the tension in the lower chord panels for a uniform live load covering the upper chord.

Here a full panel load, $p$, is held up at the intermediate apices D, F and $D^1$ and $\frac{3}{4}\,p$ at the end apices B and $B^1$ (why ?) It is convenient in taking moments, to regard the load at B as replaced by $p$ acting down and $\frac{1}{4}\,p$ acting up.

To find the chord stress $= t$, in $E^1\,C^1$, take $D^1$ as a centre of moments. Call $m$ the number of panel lengths and fractions from A to 1 and $m^1$ the number from 1 to $A^1$. Thus, $m = 3\frac{1}{2}$, $m^1 = 1\frac{1}{2}$, for centre of moments at $D^1$. The distance $A\,1 = m\,d$ and $A^1\;1 = m^1\;d$, where $d$ is a panel length. The height of truss is $h$.

We have for the left reaction,

$$R = (m + m^1 - \tfrac{1}{2})\,\frac{p}{2}$$

and its arm about $D^1$ is $m\;d$. The resultant of the forces acting down to left of $D^1 = (m - \frac{1}{2})p$, and its arm is $\frac{1}{2}\,(m + \frac{1}{2})\,d$.

The force $\frac{1}{4} p$ acting up at B has an arm $(m - \frac{1}{2}) d$.

The sum of the moments of these forces about $D^1$ equals $t h$.

$$\therefore t h = R m d + \tfrac{1}{4} p (m - \tfrac{1}{2}) d - (m^2 - \tfrac{1}{4}) \frac{d p}{2}$$

$$\therefore t = m m^1 \frac{p d}{2 h} \ldots\ldots\ldots (13)$$

29. The stress in any upper chord panel is the mean of the stresses in the two adjacent lower chord-panels. To prove this, let Q = resultant of loads to left of $E^1$ and $x$ its distance from F; then the moment of all forces to left of $E^1$, about F, divided by $h$ gives,

The stress in E E′ = (R. H F − Q $x$) ÷ $h$

Similarly the stress in E′ C′ is

$$[R. H D' - Q (x + d)] \div h$$

The mean of these two stresses is,

$$[R. A E' - Q (x + \tfrac{1}{2} d)] \div h,$$

which is seen to be the moment of all forces to the left of E′, about E′, divided by $h$, and hence this mean is the stress in F D′. Q. E. D.

30. *Example I.*—In the 5-panel deck truss of fig. 9, let $d = 16$, $h = 10$ feet and the live load on one truss, 2000 pounds per linear foot, corresponding to a very heavy train of railroad cars. This gives $p = 32{,}000$ lbs., and

$$\frac{p\,d}{2\,h} = 25{,}600.$$

Using formula (13) we find the stresses in A C, C E, and E E', respectively to be 57,600, 134,400 and 160,000 pounds. The mean of the first two, 96,000 lbs., is, by Art. 29, the stress in B D, and the mean of the last two, 147,200 lbs., is the stress in D F.

Test these results by the method of chord increments.

*Example II.*—For a railroad bridge, the weight per linear foot for the above truss of 80 ft. span, by eq. 4, Art. 5, is 580, giving a weight per panel of 9280 pounds. This weight should be used in practice, but, for convenience of computation, let us take a panel weight of 9000 lbs. and express it in thousand pounds. That is, the weight per panel is 9 thousand pounds, of which one-third, or 3, will be the apex load on the lower chord and 6 on the upper chord, except at the ends, B and $B_1$, where the apex load will be assumed $\frac{3}{4}\,6 = 4.5$.

Draw the figure of the truss fig. 9 and mark on the diagonals the resulting shears : A B = 19.5, B C = 15, C D = 12, D E = 6, and E F = 3;

then by the method of chord increments, Art. 22, find the chord stresses:

| | |
|---|---|
| A C = 15.6, | B D = 27.6 |
| C E = 37.2, | D F = 42.0 |
| E E′ = 44.4, | |

Test these stresses by the method of moments. Thus for C E, centre of moments is at D and R = 19.5; therefore, stress in C E equals

$$\frac{19.5 \times 24 - 4.5 \times 16 - 3 \times 8}{10} = 37.2$$

*Example III.*—Find the maximum and minimum stresses in the diagonals of the same truss, due to the dead load above and a live load of 32 thousand pounds per panel moving over the top chord. The shears due to dead load are found as in the last example and entered in the following table:

For live load shears proceed as in Art. 15, noting that the apex loads at B and B′ are each $\frac{3}{4}(32) = 24$. The left reaction R for full load is thus 72, and this is the maximum shear in A B.

The max. shear for B C and C D equals 72, minus the reaction $(24 \times \frac{4.5}{5} = 21.6)$ due to load at B removed, and so on (see Art. 15).

| Live Load from | Members | Dead Load Shear | Live Load Shear | Total Shear. | |
|---|---|---|---|---|---|
| H′ to B | A B | + 19.5 | + 72.0 | + 91.5 | *c* |
| H′ to D | B C | + 15.0 | + 50.4 | + 65.4 | *t* |
| H′ to D | C D | + 12.0 | + 50.4 | + 62.4 | *c* |
| H′ to F | D E | + 6.0 | + 28.0 | + 34.0 | *t* |
| H′ to F | E F | + 3.0 | + 28.0 | + 31.0 | *c* |
| H′ to D′ | E′ F | — 3.0 | + 12.0 | + 9.0 | *t* |
| H′ to D′ | D′ E′ | — 6.0 | + 12.0 | + 6.0 | *c* |
| H′ to B′ | C′ D′ | — 12.0 | + 2.4 | — 9.6 | *c* |
| H′ to B′ | B′ C′ | — 15.0 | + 2.4 | — 12.6 | *t* |
| H′ to H′ | A′ B′ | — 19.5 | 0.0 | — 19.5 | *c* |

The length of a diagonal is $\sqrt{8^2 + 10^2} = 12.8$; hence we multiply each of the shears above by *sec.* $i = 12.8 \div 10 = 1.28$ to get the max. and min. stresses below (see Arts. 16 and 17), which are given in pounds.

| | A B | B C | C D | D E | E F |
|---|---|---|---|---|---|
| Max. | — 117,120 | + 83,712 | — 79,872 | + 43,520 | — 39,680 |
| Min. | — 24,960 | + 16,128 | — 12,288 | — 7,680 | + 11,520 |

In this table, the plus sign indicates tension; the minus sign, compression. It will be noticed that only D E and E F (and of course D′ E′, E′ F′) need counter-bracing.

31. *Through Warren Girder.*

Let us next consider a Warren truss of five panels, lettered as in fig. 9, but with the roadway on the bottom chord. The members A H, H B, A′ H′, and H′ B′ are now omitted. No span or height is given in this example.

If the total dead load on one truss is W, then the dead load per panel W ÷ 5 will be called $w$. The live load per panel will be called $p$. Let us suppose that the dead panel load is divided between the two chords, so that $\frac{3}{4}\,w$ rests at each lower apex and $\frac{1}{4}\,w$ at each upper chord apex, except at B and B′ where $\frac{1}{8}\,w$ is supposed to rest.

We have now, for dead load, R = 2 $w$ from which, by subtraction, we get the dead load shears on the diagonals, which can be marked on them. Then compute the live load shears* and enter both in the following table. Call $s$ = secant of the angle and $t$ = tangent of the angle, any diagonal makes with the vertical. Any

* The live load shears are most conveniently found by aid of formula (14) of the next article.

shear on a diagonal multiplied by $s$ gives its stress.

To find the chord stresses, apply the method of chord increments, Art. 22, and place the results for dead load and also for a full live load in the following table:

| Live Load from | Member. | Stress | |
|---|---|---|---|
| | | Live Load. | Dead Load. |
| A′ to C | A B | $-2.0\ p\ s$ | $-2.000\ w\ s$ |
| A′ to C | B C | $+2.0\ p\ s$ | $+1.875\ w\ s$ |
| A′ to E | C D | $-1.2\ p\ s$ | $-1.125\ w\ s$ |
| A to C | C D | $+0.2\ p\ s$ | $-1.125\ w\ s$ |
| A′ to E | D E | $+1.2\ p\ s$ | $+0.875\ w\ s$ |
| A to C | D E | $-0.2\ p\ s$ | $+0.875\ w\ s$ |
| A′ to E′ | E F | $-0.6\ p\ s$ | $-0.125\ w\ s$ |
| A to E | E F | $+0.6\ p\ s$ | $-0.125\ w\ s$ |
| A′ to A | A C | $+2.0\ p\ t$ | $+2.000\ w\ t$ |
| " | C E | $+5.0\ p\ t$ | $+5.000\ w\ t$ |
| " | E E′ | $+6.0\ p\ t$ | $+6.000\ w\ t$ |
| " | B D | $-4.0\ p\ t$ | $-3.875\ w\ t$ |
| " | D F | $-6.0\ p\ t$ | $-5.875\ w\ t$ |

+ Indicates tension. — Compression.

The shear on C D, when live load extends from A to C, is the same as the shear on C′ D′ when the load extends from A′ to C′. Similarly the second of the live

load stresses for D E and E F were computed for D′ E′ and E′ F′ for load extending from A′ to C′ and A′ to E′, respectively.

The maximum stresses on A B, B C and the chord members are found by adding the stresses due to live and dead loads; the minimum are due to dead load alone.

For the other members we find the maximum stresses by adding the upper expressions in the table; the minimum by combining the lower, which pertain to any member.

When the span and height are given together with $w$ and $p$, the stresses can all be obtained.

The results in the above table refer to any 5 panel Warren through bridge, and the table is a type of a kind prepared in bridge offices, for instant use, for all kinds of trusses and any usual number of panels.

### Shear Formulas.

32. A formula for shears for trusses of the type of fig. 10, for a uniform live

load over the bottom chord is easily derived.

Call $p$ = panel live load for one truss,
$d$ = length of panel in feet,
N = total number of panels,
$n$ = number of panels from A to the front apex load ($n = 5$ in figure).

By the method of "apex loads," which will first be given, the uniform load, half a panel length either side of an apex, will be regarded as concentrated at that apex.

This is true for an apex in the middle of the load, but it is impossible at the left end. Thus at apex $b$, the whole of the half panel of load to left of $b$ is supposed borne at $b$, whereas part of it is held up at $a$ and the true shear in panel 5 is equal to the reaction of the load, as it rests on the bridge, less this part borne at $a$. In order that a full panel load be borne to $b$, the live load must extend to $a$; in which case the true shear is equal to the left reaction, minus the half panel load borne by the stringers to $a$.

The customary method of apex loads

thus always gives an excess, except at the end panel (1) where, for full load, the half panel of load next $a$ (now coinciding with A) is borne directly by the abutment and the shear is seen to be the same by either method.

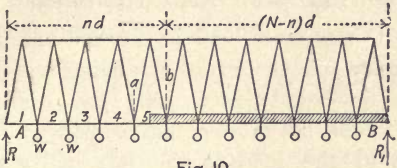


Fig. 10.

By the method of apex loads, there are $(N-n)$ loads $p$ at the lower apices, and this is true no matter where $b$, is placed. Their resultant acts at $\frac{1}{2}$ B $a = \frac{1}{2}(N-n+1)\,d$, from the right abutment B, wherever $b$ is placed; hence taking moments about B, we have,

$$R\,N\,d = (N-n)\,p\,\tfrac{1}{2}(N-n+1)\,d$$

where the left reaction = R.

Therefore, as the shear S in the $n^{th}$ panel is equal to R, we have,

$$S = (N-n)(N-n+1)\frac{p}{2N} \quad \ldots\ldots(14)$$

If we pass a section cutting either of the diagonals over the $n^{th}$ panel and the two chords, the shear S by Arts. 9 and 11, is numerically equal to V, the vertical component of the stress in either diagonal.

### 33. Exact Formula for Shears.

Let the live load in fig. 10 extend a distance $x$ to the left of $b$, $x$ being less than $d$. For convenience, call the distance B $b = (N - n)\ d = c$, and consider a thin slice of the load at the left end whose weight = 1 pound. The left reaction due to this is, $1\left(\frac{c+x}{l}\right)$, $l$ being the span, and the amount of this 1 lb. held up at $a$ is, $\frac{x}{d}$; hence the shear in panel $a\ b$ due to it is,

$$\frac{c+x}{l} - \frac{x}{d} = \frac{c}{l} - \frac{(l-d)\ x}{d\ l}$$

As $x$ increases from zero, this remains plus until it attains the value zero, for which

$$x = \frac{c}{N-1}$$

and for $x$ greater than this value the shear is minus. Hence no part of the live load

must extend to the left of $b$ a greater distance than the value just found for $x$, or the shear will be diminished.

The distance $x$ above, for maximum shear, can be easily laid off with dividers on a drawing of the truss, as follows: Divide a panel length into $(N-1)$ parts and call the length of one part $\left(\frac{d}{N-1}\right) = a$; then for $c = d, 2d, 3d, \ldots, (N-1)d$, $x = a, 2a, 3a \ldots, d$, which can be easily laid off. It is seen that $x$ increases from $x = a$ for $c = d$, to $x = d$ or a full load covering the bridge, for the end panel at the left.

Supposing that $x$ has the value found above for maximum shear, we have the exact shear $S_1$ in panel $n$ or $b\,a$, for the uniform load $w$ per linear foot, extending a distance $c + x$ from the right abutment, equal to the left reaction minus the load borne at $a$, or,

$$\frac{w(c+x)^2}{2Nd} - \frac{wx^2}{2d} = \frac{w}{2Nd}\left[\frac{N^2c^2}{(N-1)^2} - \frac{Nc^2}{(N-1)^2}\right]$$

$$\therefore S_1 = \frac{w}{2d}\,\frac{c^2}{N-1} = \frac{w}{2(N-1}\,(N-n)^2 \ldots (15)$$

In this formula $w\,d = p$ = live panel load and $(N - n)$ is the number of panel lengths from $b$ to B. Calling $(N - n) = m$, (15) can be written,

$$S_1 = \frac{m^2}{N-1}\frac{p}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

34. Let us compare the results by (14) and (15) for the five panel Warren truss of Art. 31:

| | S by (14) | S by (15) |
|---|---|---|
| $n = 1$ | 2.0 $p$ | 2,000 $p$ |
| $n = 2$ | 1.2 $p$ | 1.125 $p$ |
| $n = 3$ | 0.6 $p$ | 0.500 $p$ |
| $n = 4$ | 0.2 $p$ | 0.125 $p$ |

The difference in the shears by the two methods is seen to be greatest at the centre panel, where it amounts to 0.1 $p$. This difference increases slightly with the number of panels, so that for a 12 panel truss it attains the value 0.114 $p$. It is, therefore, small for any usual value of N. The approximate method is thus seen to give a slight excess of shear in the interior panels which attains its maximum at the centre,

where the section of the web members is small and a slight excess is not objectionable.

35. *Formula for the exact shear for a Deck Warren truss.*

If in Fig. 9, Art. 28, we suppose the uniform load $w$ per foot to rest on the top chord a distance $c + x$ from H′ where $c = m\,d$, is the distance from H′ to an apex and $m$ the number of panels and fractions in this distance, the reasoning of Art. 33 leads to the same value of $x = \frac{c}{N-1}$ and the same formula for $S_1$ in terms of $c$, but not in terms of $n$.

$$\therefore S_1 = \frac{w}{2\,d}\,\frac{c^2}{N-1} = \frac{m^2}{N-1}\,\frac{p}{2} \ldots\ldots (17)$$

36. *Example.*—Show, by a procedure similar to that of Art. 27, that the second differences derived from the terms given by formulas (14), (15) and (17) are all constant, and hence can be used as a check on computation.

37. The formulas (14), (15) and (16), for a *through* bridge, give the shear in the web members over the panel $a\,b$, fig. 10,

whether their inclinations to the vertical are the same, as in a Warren truss, or different as in Howe, Pratt and other trusses; for the inclination of the web members did not enter into the derivation of the formulas. Hence we may conceive the two web members over the panel *a b*, fig. 10, to have their tops moved forwards or backwards, so that one of them becomes vertical, giving trusses of the Pratt or Howe types (Art. 25), and the shears for live load in these are equal and are given by eq. (14) by the method of apex loads, and by eqs. (15) or (16) by the exact method.

Formulas (14) and (17) apply to Pratt and Howe trusses when the live load passes over the top chord. For such *deck trusses*, to find the position of the live load giving maximum shear in any tie or strut, the simple rule is, pass a section cutting only the two chords and the web member (never more than three members in action in all), and move the live load from the right abutment up to the apex just right of the section, if the method of apex loads is to

be used, or a distance $x = c \div (N - 1)$ beyond this point if the exact method is to be used. No apex load should ever be placed to the left of the section, or the shear is diminished.

This rule, for deck trusses and for apex loads, requires that for maximum shear in a vertical post of a Pratt truss, the front apex load must be directly over it, whereas for a Howe truss the vertical tie is most strained when the front apex load is one panel length to its right.

The Pratt and Howe Bridges, both through and deck, will now be taken up in detail.

### Through Pratt Truss.

38. Let figure 11 represent a through Pratt truss of 120 feet span, divided into 6 panels of 20 feet each. The height, centre to centre of chords, is assumed at 20 feet. The dead load at each lower apex is assumed, for convenience in computation, at 6, at each upper apex 3 and the live panel load at 18, all loads being expressed in thousand pounds. The dead

load reaction is 22.5 and we get the shears due to dead load, in the diagonals over panels 1, 2, 3 ....., by the continued subtraction of 9.

Selecting the method of "apex loads," formula (14) gives shear equal to

$$(6 - n)(7 - n) \times 1.5$$

From this, the live load shears are computed below for $n = 1, 2, \ldots\ldots$, and added to the dead load shears.

| Front of load at | $n$ | Dead load shear | | Live load shear | | Total. |
|---|---|---|---|---|---|---|
| C, | 1, | 22.5 | + | 45 | = | 67.5 |
| E, | 2, | 13.5 | + | 30 | = | 43.5 |
| G, | 3, | 4.5 | + | 18 | = | 22.5 |
| E′, | 4,— | 4.5 | + | 9 | = | 4.5 |
| C′ | 5,— | 13.5 | + | 3 | = | −10.5 |

Find the first and second differences of the total shears, and notice that the second differences are each equal to 3, a constant, which verifies the work.

The dotted lines in the two central panels represent counter ties and by Art. 25, are not in action when the main diagonals (ties) are in action. We shall now

take up, in some detail, the maximum and minimum stresses ever experienced by the web members.

*Batter brace* A B. The max. shear is for full load and is 67.5; the min. shear is due to dead load only and is 22.5

*Hip vertical* B C. Its max. stress is simply one panel of live load and the dead load held up at C or 6 + 18 = 24; min. stress = 6.

*Main tie* B E. The load extends from A′ to E. The max. shear is 43.5 and the piece is in tension, as the shear is plus (see fig. 4, Art. 11).

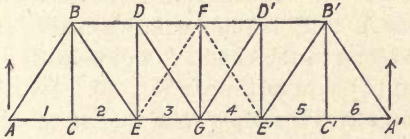


Fig. II.

*Main Tie* D G.—For max. shear, the load extends from A′ to G, giving a shear + 22.5; hence the tie D G is in action (Art. 11) and the counter E F is out of action (Art. 25).

*Post* D E.—The post D E is subjected to a stress equal to the vertical component of the stress in D G plus the dead load held at D. Therefore, it receives its maximum stress when the shear in D G is a maximum, or for live load from A′ to G ∴ max. stress in post D E = 22.5 + 3 = 25.5.

*Counter* F E′.—With live load from A′ to E¹, the shear is + 4.5, and therefore acts up; hence, if we pass a section, etc., as in Art. 11, V acts down; hence the counter is in action and the main tie G D′ out of action, as it cannot take compression. The max. shear in the counter is thus 4.5.

*Post* F G.—The max. stress in F G for load from A′ to E′ is the vertical component of the stress in the counter (4.5) plus the dead load (3) held at F or in all, 7.5. Its minimum stress is simply that due to the dead load resting at the top = 3, when the counters meeting there are both out of action, which is the case when there is no live load or a full live load on the bridge (why?)

It may be remarked that, when the live load extends from A′ to E′, giving the shear in panel 4 = + 4.5, the shear in

panel 3 is greater by the dead load held up at F and G; therefore the shear in panel 3 = + 4.5 + 9 = + 13.5, and since it is plus, D G is in action and E F is not in action: so that the counter E F does not transmit any stress to post F G for this position of the load. Further, this position of the load (from A′ to E′) gives the maximum stress in both E′ F and F G, for if any load is taken off or any added to left of E′, the positive shear is diminished (Art. 13); in fact, if an apex load is added at G, the shear in panel 4 = shear in panel 3 minus the loads (9 + 18) held at F and G = 22.5 — 27. = — 4.5 and the counter E′ F is thus out of action (the shear being minus), and F G sustains only the dead load at its top.

*Post* D′ E′, with the live load extending from A′ to E′, we have just seen that counter E′ F is in action; hence G D′ is out of action, and cannot transmit any stress to post D′ E′, which thus sustains only the dead load 3, at its top. This is the *minimum* stress in D′ E′, and consequently the minimum stress in post D E for a live load

from A to E is the same or 3 thousand pounds.

*Tie* B′ E′.—With one apex live load at C′ the shear in E′ B′ = — 13.5 from dead load + 3 from live load = — 10.5 as given above. If any live load is added or taken off, the + shear is less (Art. 13) which would increase the minus shear (— 10.5) ; hence this is the smallest negative shear that E′ B′ can sustain, and it causes tension in E′ B′. Let the reader draw the truss from A up to a section cutting E′ C′, E′ B′ and D′ B′ and supply forces as in fig. 3 (Art. 9) equal to the forces exerted by the part of the truss right of the section upon the cut members. Then since the shear from the left is minus 10.5, it acts down, therefore the supplied force acting on E′ B′ at the section, acts up, since its vertical component acts up, hence E′ B′ is in tension. This minimum tension in E′ B′ for an apex load at C′ is also the minimum tension in B E for an apex load at C.

If a counter extended from C′ to D′, it could never be in action for the panel loads given, as we have just seen that E′

$B^1$ is always in action for any position of the live load, since the shear on it is always minus.

The rule therefore is, *never insert a counter in a panel where the shear is always minus.*

A counter is required in the 4th panel for live load from A′ to E′. Similarly a counter is required in the 3d panel for a live load from A to E (its maximum shear being 4.5), and no others are needed.

On multiplying the max. and min. shears above, on the inclined web members, by *sec.* $i$ = *sec.* 45° = 1.414, their stresses are obtained.

The maximum and minimum stresses in all the web members are given in the following table in pounds, + indicating tension, — compresssion:

| | AB | BC | BE | DE | DG | EF | FG |
|---|---|---|---|---|---|---|---|
| Max. | —95445 | +24000 | +61509 | —25500 | +31815 | +6363 | —7500 |
| Min. | —31815 | + 6000 | +14847 | — 3000 | 0 | 0 | —3000 |

The chord stresses are a maximum for a full live load. Therefore, by formula (12)

Art. 26, since $P = 9 + 18 = 27$, $d = 20$, $h = 20$,

$$t_n = c_n = n\,m \times 13.5$$

In this formula, $n$ is the number of panel lengths from A to the centre of moments for the chord-piece considered, and $m = 6 - n$ is the remaining number of panels. The chord stresses in pounds are as follows:

A C E, $n = 1$, 67,500
B D or E G, $n = 2$, 108,000
D F, $n = 3$, 121,500

The minimum chord stresses due to dead load only are $\frac{9}{27}$ of the above.

*Example.*—Compute the stresses in all the members of a through Pratt truss of 140 ft. span divided into 7 panels, and with a height of 26 ft. 8 inches, for a dead load of 10,500 lbs. per panel (of which $\frac{2}{3}$ is supposed to be borne by lower apices and $\frac{1}{3}$ by upper apices) and a live panel load of 21, 600 pounds. Use the method of apex loads. The dead panel load by formula (2), Art. 5, is found to be 10,440 for a width of bridge of 24 feet. The live-panel load allowing 90 lbs. per sq. ft. of floor is $24 \times 20 \times 90 \div 2 = 21{,}600$. The loads assumed are thus seen to be usual ones. Determine where counters should be used.

## Through Howe Truss.

39. (See figure 12.) Take the same span, height, number of panels, dead and live loads as in the bridge first examined in Art. 38. The shears are as there given. Read the note on counters in Art. 25. As the shear is minus (— 10.5), first in the 5th panel, no counter is needed there, as the main brace then is in action (under its minimum stress). Hence counters were omitted from panels 2 and 5, though in practice they are put in wooden Howe bridges in every panel to keep the main braces in place, and also as some support to them from lateral flexure.

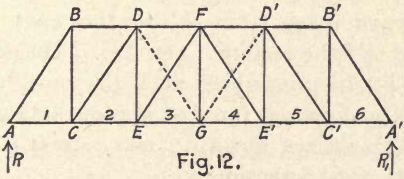

Fig. 12.

By Art. 12 we have the rule: when the shear on a diagonal is *plus*, if the top of the diagonal leans *away from* the left

abutment the diagonal is in compression; if the shear is *minus* and the top leans *towards* the left abutment the diagonal is again in compression.

Hence passing sections cutting each diagonal in turn; we have

Live load from

A′ to C, max. shear in A B = + 67.5
A′ to E, " " " C D = + 43.5
A′ to G′, " " " E F = + 22.5
A′ to E′, " " " D′ G = + 4.5
A′ to C′, min. shear in C′ D′ = — 10.5

By the rule above, all of these diagonals are in compression for the loading assumed.

*Vertical B C.*—Pass a section cutting A C, B C and B D. For max. shear, the live load extends from A′ to C, as it must always reach from A′ to the apex just right of the section (Art. 13). The shear is + 67.5 minus load at B (3) = + 64.5, hence supposed force at section acting on B C must act down, ∴ max. stress in tie B C = 64.5 tension.

*Vertical* D E.—Section must cut C E, D E, D G and D F. Is the counter in action? With load from A′ to E, shear in

C D is + 43.5 and in panel 3, (43.5 — 27), a plus quantity ∴ E F is in action and D G out of action. Hence the section cuts only three members in action and the stress in D E is equal to the shear in it, or 43.5 — dead load at D (3) = 40.5 tension.

*Vertical* F G.—With a live load from A′ to G, the shear in panel 3 is + 22.5 and in panel 4, + 22.5 — 27 = — 4.5; hence F E and F E′ both suffer compression and the counters D G and D′ G are not in action. Therefore F G holds up only a lower panel of dead load and one of live load; hence max. tension in F G = 6 + 18 = 24.

F G must also be tested with a live load from A′ to E′, which gives a shear in panel 4 of + 4.5 and a shear in panel 3 of + 4.5 + 9 = + 13.5, for which E F and G D′ are in action and D G and F E′ out of action. A section can then be passed through E G, F G and F D′ (alone in action on the section) and the stress in F G is seen to be equal to the shear in E F (+ 13.5) minus the load at F (3) or 10.5; or it is equal to the shear in G D′ (+ 4.5)

plus the dead load at G (6) = 10.5. This is less than the 24 above. The *minimum* stress on F G is for dead load only. The counters are then out of action and F G sustains only the dead load at G = 6.

For *minimum* stress on *vertical* D′ E′ the live load extends from A′ to C′. The shear in panel 5 = — 10.5; in panel 4, — 10.5 + 9 = — 1.5, ∴ D′ C′ and F E′ are both in action and G D′ out of action. Hence a section can be passed, as in fig. 13, cutting only three members in action. The shear in D′ E′ = — 10.5 + 3 (load

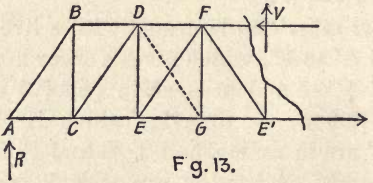

Fig. 13.

at D′) = — 7.5. As it acts down, V must act up, giving a tension in D′ E′ = 7.5. This is its minimum stress, as the live load covers the shorter segment.

The minimum stress in D E is conse-

quently 7.5 tension when the live load extends from A to C.

It is well to remark that, with other panel loads than the ones assumed, it could very well happen that for a live load from A′ to C′, the shear in panel 5 might be minus and in panel 4, plus; in which case D′ C′ and D′ G are in action, whence F E′ is out of action and D′ E′ sustains only the dead load G at E′, which in this case would be its minimum tension.

The *minimum* shear in D′ C′ = — 10.5 for live load from A′ to C′; hence the minimum compression in D C is 10.5 *sec i* for a live load extending from A to C. The *minimum* tension in B C = 19.5 and the *minimum* compression in A B is due to a shear 22.5, both for dead load alone.

The maximum and minimum stresses in pounds on the web members are then as follows:

| | A B | B C | C D | D E | E F | D G | F G |
|---|---|---|---|---|---|---|---|
| Max. | —95445 | +64500 | —61509 | +40500 | —31815 | —6363 | +24000 |
| Min. | —31815 | +19500 | —14847 | + 7500 | 0 | 0 | + 6000 |

The chord stresses are found as in the preceding article from the formula,

$$t_n = c_n = n\,m \times 13.5;$$

or expressing them in pounds, and remembering that $n$ is the number of panel lengths from A to the center of moments for the chord piece considered, we have the maximum stresses:

A C or B D, $n = 1$, . . . 67,500 lbs.
C E or D F, $n = 2$, . . . 108,000 lbs.
E G, $n = 3$, . . . 121,500 lbs.

The minimum stresses due to dead load only are $\frac{9}{27}$ of these.

*Example.*—Consider a through Howe truss of the dimensions and loading given in the example at the end of Art 38. Ascertain where counters should be used and the stresses in all the members.

## Deck Pratt Truss.

40. Let figure 14 represent a Pratt truss of 126 feet span, divided into 7 panels each 18 feet in length. To ensure counters in the three central panels, a large live load of 2,000 lbs. per ft. on one truss will be assumed to pass over the upper chord. Expressed in thousand pounds, this gives a panel load of $p = 2 \times 18 = 36$. Apex

loads of 6 and 3 will be assumed at the upper and lower chords respectively to represent the dead load.

An illustration will now be given of the working of "the exact method" for shears, Art. 33; this will not be found quite so simple as the method of "apex loads."

Equation (15) reduces in this case to $S_1 = 3\ (7 - n)^2$; and it must be remembered that the live load extends from A′ to the apex just right of the section taken, a distance, $c = md$ and in addition, $x = \frac{c}{N-1} = \frac{1}{6}\ c$ to the left of the apex.

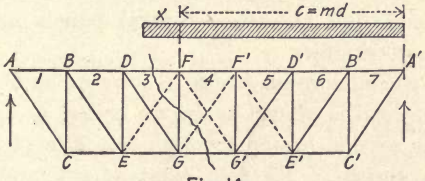

Fig. 14.

The dead-load reaction $= 3 \times 9 = 27$, from which we find dead-load shears as usual. On adding them to the live-load shears computed from the formula above,

we get the totals given below for the diagonals:

| | | | |
|---|---|---|---|
| $c = A'B$, | $n = 1$ | $S = 108 + 27$ | $= + 135$ |
| $c = A'D$ | 2 | $75 + 18$ | $= + 93$ |
| $c = A'F$ | 3 | $48 + 9$ | $= + 57$ |
| $c = A'F'$ | 4 | $27 + 0$ | $= + 27$ |
| $c = A'D'$ | 5 | $12 - 9$ | $= + 3$ |
| $c = A'B'$ | 6 | $3 - 18$ | $= - 15$ |
| $c = 0$ | 7 | $0 - 27$ | $= - 27$ |

The maximum shears in the diagonals A C, B E, D G, F G′ and F′ E′ are given by the values of S above for $n = 1, 2, \ldots, 5$. As the shear is minus in the sixth panel, no counter is required there or in the 2d panel; but the three central panels must have counters.

For maximum stress in a post, as F G, the load is placed as drawn in fig. 14, and the section passed through the five pieces as shown. To test what pieces are in action we must know, where counters are cut by the section, the amount of live load held up at the apex just right of the section. Thus, for a load 2 thousand pounds per foot, F in the figure holds up part of

the load $2x$ to left of F and a half-panel load from the right, or

$$18 + 2x - \frac{2x^2}{2d} = 18 + 6m - \tfrac{1}{2}m^2,$$

since $x = \frac{1}{6}c = \frac{1}{6}md$.

From this formula we find for

C = A′ D, live load held at D = 35.5
C = A′ F, " " " " F = 34.0
C = A′ F′, " " " " F′ = 31.5
C = A′ D′, " " " " D′ = 28.0

For brevity, the shear on a diagonal in panel 4 for a certain position of the load will be called $S_4$, and *for the same position of the load* the shears on diagonal in panels 3 and 5 will be called $S_3$ and $S_5$ respectively. Similarly for any panel. Also observe that the shear on F G, at the section, fig. 14, when E F and G F′ are out of action, is equal to the left reaction minus all the loads up to the section, or, in this case, to $S_3$ — load at G = 57 — 3 = 54.

Bearing in mind the rules of Art. 12 and what precedes, the reader should find no difficulty in following the brief consideration of posts that follows.

*Post* B C; $c$ = A′ B; max. stress in B C = $S_1 - 3 = 135 - 3 = 132$.

*Post* D E; $c$ = A′ D; $S_2 = 93$, $S_3 = 93 - (35.5 + 9)$ or plus, ∴ D G is in action and E F not in action, ∴ max. stress in D E = $93 - 3 = 90$.

*Post* F G; $c$ = A′ F; $S_3 = 57$, ∴ E F out of action. $S_4 = 57 - (34 + 9)$ or plus ∴ G F′ is out of action and the section cuts only three pieces in action, ∴ maximum stress in F G = $S_3 - 3 = 57 - 3 = 54$.

*Post* F′ G′.—The min. stress is for dead load only, when none of the (dotted) counters act, and is equal to dead load at F′ = 6. The same holds for F G.

*Post* D′ E′. For min. stress, try $c$ = A′ B′ and pass a section through G′ E′, F′ E′, D′ E′ and D′ B′. $S_6 = -15$; to find $S_5$ we first find the live load held up at D′ =

$$\frac{w x^2}{2 d} = \frac{2 \times 3^2}{2 \times 18} = 0.5$$

∴ $S_5 = S_6 +$ loads at D′ and E′ $= -15 + 9 + .5 = -5.5$, ∴ G′ D′ is in action and F′ E′ out of action; hence the section

cuts but three members in action and the live load covers the smaller segment of the span ∴ the shear in Post D′ E′ is a minimum and is equal to $S_6$ + load at E′ = − 15 + 3 = − 12.

[For other loads than those given, it might happen that F′ E′ as well as E′ B′ was in action; in this case E′ D′ supports only the loads at its top, or 6 + amount of live load held at D′.]

The minimum stress in post D E when $c$ = A B is thus, for the loads first taken = 12. For $c$ = A′ B′, we have smallest shear in E′ B′ = $S_6$ = − 15; this is also equal to the smallest shear in B E for $c$ = A B.

Under dead load only, A C receives a shear of 27 and B C of 27 − 3 = 24. The diagonals in all counter-braced panels have a minimum stress zero, for when one is in action the other is out of action, and vice versa.

*Example.*—Compute the maximum and minimum shears in the web members of the truss just considered, by the method of apex loads, and compare the results with those just obtained.

41. *Deck Howe Truss.*—The investigation of the trusses previously given should enable the attentive student to place the loads for maximum and minimum shears, and draw the proper sections for a deck Howe truss. Assume the truss of 8 panels with a dead apex load on the upper chord of 7, and on the lower chord of 3, the live apex load on upper chord being 16; all loads being in thousand pounds. Letter the truss as in fig. 12, Art 39, calling the centre vertical H I. For a deck truss, vertical posts should be placed at A and A′, fig. 12, to meet the upper chord produced. The additional pieces are not parts of the truss proper. Use the approximate method of apex loads, Art. 32, and attend strictly to the principles of arts. 12 and 13. The sections taken must never cut more than three pieces in action.

The maximum and minimum *shears* for the braces are as follows:

| | AB | CD | EF | GH | FI |
|---|---|---|---|---|---|
| Max. | 91 | 67 | 45 | 25 | 7 |
| Min. | 35 | 23 | 9 | 0 | 0 |

The stresses for the vertical ties are :

| | B C | D E | F G | H I |
|---|---|---|---|---|
| Max. | 70 | 48 | 28 | 10 |
| Min. | 26 | 12 | 3 | 3 |

For $d = 20$, $h = 26$, we have the following maximum chord stresses:

A C, B D, stress = 70,
C E, D F, stress = 120,
E G, F H, stress = 150,
G I, stress = 160.

42. *Remark on Counters.*—When a Pratt bridge with counters is first swung clear of false-works, the main ties, but not the counters, are in action. If a light load is put on, and the counters made taut; for a heavier or full load they will be loose and for the dead load only, under slight initial stress. This is because the loading causes increased deflection of the truss, involving lengthening of main ties and shortening of counters.

If an initial stress is put in the counters, with no live load on bridge, the main ties have an increase of tension equal to this stress. Thus in fig. 14, Art. 40, we suppose

an initial stress in counter E F, whose vertical component is V′, and conceive the pieces E G, D G, E F and D F severed by a plane. On supplying forces at the cut parts equal to the action of the right part of the truss upon the cut members, we have, as in arts. 9—11, $R - \Sigma w + V' = V$, since V′ acts up and V down; or the vertical component V, of the stress in D G has been increased by V′; hence the stress in D G has been increased by the initial stress (V′ *sec i*) in the counter E F.

If the initial stress in E F, and the increase due to it in D G, are resolved into vertical and horizontal components at D, E, F and G, they balance along posts and chords, and do not affect any other panel.

### The Whipple Truss.

43. For long spans, over 180 feet say, the inclination of the diagonals, for usual panel lengths and heights, of the trusses previously examined, is not the best for economy. Hence, the use of multiple systems having the ties cross two or more panels, as in the once popular Whipple truss, fig. 15.

In the truss of 12 panels shown, we shall first consider the dead load per panel $w$, as concentrated at the lower apices, and afterwards regard it as divided in the usual proportion between the two chords. The loads $w$ are represented by circles placed below the chord and the live load per panel $p$, by rectangles placed just above the lower apices. It will be noticed that any weight, as that at $f$, can travel to either abutment by only one web system, as $a$ B $d$ D $f$ H $h$ J $j$ L $m$. The weight at $e$ must follow the other system, $a$ B $c$ C $e$ G $g$ I $i$ K $k$ L $m$. If the counter (dotted line) is not in action, then the main tie replaces it, and vice versa. We observe,

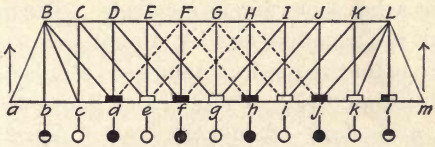


Fig. 15.

therefore, that the black weights travel towards either abutment only by the first system; the others, by the second.

The loads at $b$ and $l$ are carried to B and

L by the vertical suspenders B$b$ and L$l$, and they will be regarded as divided equally between the two systems. For the chord stresses the results are the same, whatever division of these loads at $b$ and $l$ we make between the two systems.

For the shears, if the whole of $p$ at $l$ belongs to one system, the error in the left reaction (and therefore the shear) over the above supposition is only $\frac{1}{12}\frac{p}{2}$, a very small amount. With vertical end posts, as in fig. 16, there is no uncertainty, as then any load at $b$ or $l$ can travel by one system only.

It may occur to the reader that this truss has superfluous members. Leaving out the counters, as they do not act when the main ties are in action, $m$ = number of pieces = 45, $n$ = number of joints = 24 $\therefore m = 2n - 3$, and the test of Art. 2 is fulfilled; hence the figure of the truss has no superfluous lines, but just enough sides to completely fix its form.

In finding stresses, the method used is to consider the partial truss along which

the black weights travel as acting independently of the other partial truss, and properly combine the results where the two trusses have the same member in common.

Thus, to find the shears in the diagonals and end posts for the dead loads, let us first consider the partial truss $a$ B $c$..., with loads $w$ at $c$, $e$, $g$, $i$, $k$ and $\frac{1}{2}$ $w$ at $b$ and $l$; the left reaction is thus $\frac{1}{2}$ 6 $w$ = 3 $w$. Hence shear in B $c$ = 3 $w$ — $\frac{1}{2}$ $w$ (at $b$) = $2\frac{1}{2}$ $w$; in C $e$ = $2\frac{1}{2}$ $w$ — $w$ (at c) = $1\frac{1}{2}$ $w$; in E $g = \frac{w}{2}$; in $g$ I or G $i$ (whichever is in action when live load is added) = $\frac{1}{2}$ $w$ — $w$ (at $g$) = — $\frac{1}{2}$ $w$, and so on.

For the black weight system we have similarly, reaction = $2\frac{1}{2}$ $w$; shear in B$d$ = 2 $w$; in D$f$ = 1 $w$; in F$h$ or $f$H = $o$; in H$j$ or H J = $o$ — $w$ = — $w$, and so on.

Notice that these shears are the same for the inclined web members, whether the dead load is supposed concentrated at the lower chord, as above, or divided between the two chords in any manner.

The live-load shear on a tie, when the

load extends from $m$ to its foot, is equal to the left reaction. Thus, when the load is placed as in fig. 15, we get the maximum shear on B $d$ due to the weights $p$ at $d, f$, $h$, $j$ and $\frac{1}{2}p$ at $l$, equal to

$$\tfrac{1}{12}p\,(9 + 7 + 5 + 3 + 0.5) = \frac{24.5}{12}\,p.$$

Similarly we proceed for any of the other ties or counters. The shears are as follows:

| Live-load from | Member. | Dead-load Shear | Live-load Shear |
|---|---|---|---|
| $m$ to $b$ | $a$ B | $+ 5.5\,w$ | $+ 5.5\,p$ |
| " " $c$ | B $c$ | $+ 2.5\,w$ | $+ \frac{30.5}{12}\,p$ |
| " " $d$ | B $d$ | $+ 2.0\,w$ | $+ \frac{24.5}{12}\,p$ |
| " " $e$ | C $e$ | $+ 1.5\,w$ | $+ \frac{20.5}{12}\,p$ |
| " " $f$ | D $f$ | $+ 1.0\,w$ | $+ \frac{15.5}{12}\,p$ |
| " " $g$ | E $g$ | $+ 0.5\,w$ | $+ \frac{12.5}{12}\,p$ |
| " " $h$ | F $h$ | $- 0.0\,w$ | $+ \frac{8.5}{12}\,p$ |
| " " $i$ | G $i$ | $- 0.5\,w$ | $+ \frac{6.5}{12}\,p$ |
| " " $j$ | H $j$ | $- 1.0\,w$ | $+ \frac{3.5}{12}\,p$ |
| " " $k$ | $i$ K | $- 1.5\,w$ | $+ \frac{2.5}{12}\,p$ |
| " " $l$ | $j$ L | $- 2.0\,w$ | $+ \frac{0.5}{12}\,p$ |
| " " $l$ | $k$ L | $- 2.5\,w$ | $+ \frac{0.5}{12}\,p$ |

We have assumed here that counters would only be needed where placed in the figure; hence the sum of the shears above for any member gives its greatest total shear except for the last three members, where the sum is supposed to be negative, and the total shear a minimum, as the shortest segment of the span is covered by the live load.

Of course we multiply each total shear by the secant of the angle the web piece makes with the vertical to get the corresponding stress.

The counters and the main ties, crossing at their centres, have zero for their minimum stress. The minimum stress on *a* B and *b* B is due to dead load.

Let us consider next the vertical posts. They receive the vertical components (= shear in amount) of the stress in the diagonals connecting with their tops, and in addition any part of the dead load supposed to be concentrated there. Their maximum or minimum stresses are thus the maximum or minimum shears (irrespective of sign) in the ties or counters connecting

with their tops increased by the dead-apex load on the top chord.

When the counters are out of action that meet at F, G and H (as they are for a full live load) the posts F $f$, G $g$ and H $h$ sustain only the deads loads at their tops, their minimum stresses. For such loading as causes counter G $i$ to act, $g$ I is out of action, and post I $i$ sustains only the dead load at I. Similarly the minimum stress in post J $j$ is the dead load at J, H $j$ being in action and $h$ J out of action. Similarly for E $e$ and D $d$. The posts K $k$ and C $c$ bear the minimum stress $(1.5\ w - \frac{2.5}{12}\ p) +$ dead load at top, as explained above.

The hypothesis that the two partial trusses act independently, forms the basis for the investigation of any compound or "double-intersection" system. This is not actually true, as the chords of the two systems cannot change length independently under loading; but it is probably sufficiently near for practice. For railroad bridges where actual wheel loads are used, the computation, even on this inexact hy-

pothesis, is very laborious; so that, for both reasons, specifications now give the preference to those trusses in which the stresses can be accurately computed.

The maximum stresses in the chords are for a full live load. Call $w' = w + p$; find the shears on each web member due to $w'$ at each lower apex, and apply the rule of Art. 22. It is well to mark the shears on the members, as is done in fig. 16, for a Whipple truss with vertical end posts. On decomposing the stresses in the diagonals at each end, as shown, into vertical and horizontal components, the latter give the chord increments. Thus, in fig. 16, stress

$$\text{in A B} = 3\ w' \frac{d}{h} + 2.5\ w' \frac{2d}{h} = 8\ w' \frac{d}{h}$$

$$\text{Stress in B C} = \text{stress in A B} + 2\ w' \frac{2\ d}{h}$$

$$= 12\ w' \frac{d}{h}$$

Similarly at the apices of either chord.

[Notice that the chord stress is a maximum where the shear is zero.]

For the truss fig. 15, a similar method

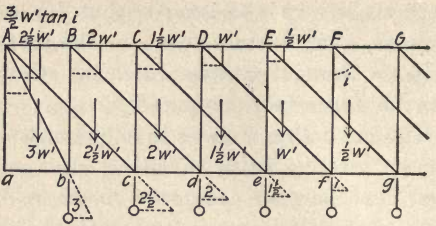


Fig. 16.

of chord increments (Art. 21) applies, giving the stress in

$$a\,b\,c \quad = \quad 5.5 \times w' \frac{d}{h}.$$

$$c\,d \quad = \quad 8. \times \text{“}$$

$$\text{B C}, d\,e \quad = 12 \times \text{“}$$

$$\text{C D}, e\,f \quad = 15 \times \text{“}$$

$$\text{D E}, f\,g \quad = 17 \times \text{“}$$

$$\text{E F}, \text{F G} \quad = 18 \times \text{“}$$

The last result can be tested by taking moments about $g$, since the counters are out of action. The reaction for the fully loaded truss is $5.5\ w'$. There are full loads at $b$, $c$, $d$, $e$ and $f$, whose resultant acts at $d$. Hence stress in E I equals $(5.5\ w' \times 6\ d - 5\ w' \times 3\ d) \div h = 18\ w'\ d \div h$, as found above.

When formulas have been found, as in this case, applicable to any Whipple truss of 12 panels, it is best, for given values of $w$, $p$, $d$ and $h$, to compute the successive results by evident successive subtractions for shears and additions for chord stresses, and check the results in the end by the formula.

*Example.*—Deduce formulas similar to those above for shears and chord stresses for a through Whipple truss of 11 panels, supposing only four counters needed, those meeting at F and G (fig. 15); also give maximum and mimimum stresses in the posts. Counters are not needed for those panels where the total shear is minus (Art. 38); but this can be ascertained only when numerical values have been inserted in the formulas. In all trusses counters are frequently inserted where not theoretically needed for the loads assumed, as still heavier live loads might necessitate their use.

## Lattice Truss.

44. The seven, panel lattice truss, fig. 17, can be divided into two systems; the one shown by full lines and the other by the dotted lines, called respectively the full and the dotted systems. The loads (live or dead) held at B, C and B′ C′,

will be divided, as before, equally between the two systems. Call $p$ the panel live-load, then the max. live-load shear in A B $= \frac{1}{2}$ 6 $p$ = 3 $p$. The other maximum live-load shears are equal to the left reactions for the full or the dotted systems for loads covering the longer segment of the span, and are as follows:

B E; $p$ at E, G′ and 0.5 $p$ at C′,
shear $= p\,(\frac{5}{7} + \frac{3}{7}) + \frac{1}{2}p \times \frac{1}{7} = +\,1.215\,p$

C D and D G; $p$ at G, E′ and 0.5 $p$ at C′,
shear $= p\,(\frac{4}{7} + \frac{2}{7}) + \frac{1}{2}p \times \frac{1}{7} = +\,0.929\,p$

E F and F G′; $p$ at G′ and $\frac{1}{2}$ $p$ at C′,
shear $= p \times \frac{3}{7} + \frac{1}{2}p \times \frac{1}{7} = +\,0.5\,p$.

For minimum shears on these same pieces, it will be simplest here to regard the live load as extending from the left abutment to the nearest apex left of the section. The left reaction, minus the loads up to the section, gives the shear

B E; $\frac{1}{2}$ $p$ at C,

$$\text{shear} = \frac{p}{2}\frac{6}{7} - \frac{p}{2} = -\,.072\,p.$$

C D and D G; $\frac{1}{2}$ $p$ at C, shear $= -\,.072\,p$.
E F and F G′; $\frac{1}{2}$ $p$ at C and $p$ at E,

Shear $= \frac{1}{2} p \times \frac{6}{7} + p \times \frac{5}{7} - 1.5\ p = -.357\ p.$

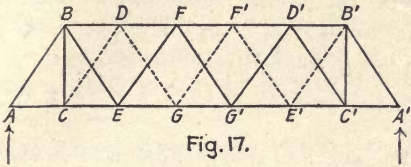

Fig. 17.

On multiplying each of these shears by *sec* A B C $= s$ and attending to rules of Art. 12, we find the stresses in the table below, corresponding to live load.

For the *dead-load shears*, $\frac{2}{3}\ w$ will be assumed as the dead load held at each lower apex and $\frac{1}{3}\ w$ at each upper apex, $w$ being the dead-panel load. The loads at B, C and B′, C′, on either the full or the dotted systems, being symmetrically placed with respect to the centre, give reactions equal $\frac{1}{2}\ w$ on each system, to which must be added reactions caused by the remaining loads. For *the full system*, we have the remaining loads $\frac{1}{3}\ w$ at F and D′, $\frac{2}{3}\ w$ at E and G′; therefore, the left reaction R equals

$\frac{1}{2} w + \frac{1}{7} [\frac{1}{3} w (4 + 2) + \frac{2}{3} w (5 + 3)] = 1.548 w$.

For *the dotted system*, the remaining loads are $\frac{1}{3} w$ at D and F′, $\frac{2}{3} w$ at G and E′; hence the left reaction R′ equals $\frac{1}{2} w + \frac{1}{7} [\frac{1}{3} w (5 + 3) + \frac{2}{3} w (4 + 2)] = 1.452 w$.

The shear in A B is the sum of R and R′, or 3 $w$, which is otherwise evident.

The shear in B E = R — loads on full system held at B and C $(.5 w) = (1.548 - .5) w = 1.048 w$. Similarly, shear on C D $= (1.452 - .5) w = 0.952 w$. Shear on E F = shear on B E — load at E $= (1.048 - .667) w = 0.381 w$.

Similarly, shear on D G $= (.952 - .333) w = 0.619$ w; shear on F G′ $= (.381 - .333) w = 0.048 w$ and shear on G F′ $= (.619 - .667) w = - 0.048 w$.

It is best, in order to avoid mistakes, to mark the stresses caused by these shears at once on a sufficiently large sketch of the truss, L for live load, D for dead load; also the character of the stress they give on each diagonal. Notice that D G and G F′ are both in tension under dead load

only, so that the chord increment at G is $(.619 - .048)\ w = 0.571\ w$.

The dead-load stress in B C equals the vertical component of the compression in C D plus the *entire* load held at C or $(.952 + .667)\ w = 1.619\ w$, as B C belongs to both systems.

The *chord stresses* due to dead load are quickly found, by the rule of Art. 22, to be for

$$A\,C = 3.000\ w\ t,\ B\,D = 4.048\ w\ t$$
$$C\,E = 3.952\ w\ t,\ D\,F = 5.619\ w\ t$$
$$E\,G = 5.381\ w\ t,\ F\,F' = 6.048\ w\ t$$
$$G\,G' = 5.952\ w\ t,$$

noting the peculiarity above at G.

The tangent of the inclination of a diagonal to the vertical $= t = d \div h$.

Finally, we have to find *the chord stresses due to a full live load.*

We proceed, as for dead load, in first getting reactions; then shears and finally chord stresses, for $p$ at each lower apex, $\frac{1}{2}\ p$ at C and C′ going to either system. The left reaction due to $\frac{1}{2}\ p$ at C and $\frac{1}{2}\ p$ at C′ is $\frac{1}{2}\ p$; therefore, left reaction for full system equals

$$\tfrac{1}{2}\,p + p\,(\tfrac{5}{7} + \tfrac{3}{7}) = 1.643\,p,$$

and for dotted system

$$\tfrac{1}{2}\,p + p\,(\tfrac{4}{7} + \tfrac{2}{7}) = 1.357\,p.$$

The sum 3 $p$ is the shear in A B. The shears in other members are then as follows: B C $= (1.357 + 0.500)\,p = 1.857p$; C D $= (1.357 - .500)\,p = .857\,p =$ D G; B E $= (1.643 - .500)\,p = 1.143\,p$; E F, F G′ $= (1.143\,p - p) = 0.143\,p$; F′ G $= (0.857\,p - p) = -\,0.143\,p$, giving tension in F′ G; also there is tension in D G, ∴ chord increment at G $= (.857 - .143)\,p = .714\,p$. The resulting stresses are given in the following table:

In this table + indicates tension, — compression. When $p$, $w$, $s$, $t$ are given the maximum and minimum stresses in all the members can be found by combining results above for live and dead loads.

The diagonals requiring counter-bracing are those whose maxima are of a different kind of stress from their minima stresses.

The truss just examined is used for riveted structures of 75 to 125 ft. spans. With vertical suspenders dropped from the

| Live Loads at | Member | Stress due to Live Load | Stress due to Dead Load |
|---|---|---|---|
| C to C′ | A B | $-3.000\,p\,s$ | $-3.000\,w\,s$ |
| E, G′, C′ | B E | $+1.215\,p\,s$ | $+1.048\,w\,s$ |
| C | B E | $-.072\,p\,s$ | $+1.048\,w\,s$ |
| G, E′, C′ | C D | $-0.929\,p\,s$ | $-0.952\,w\,s$ |
| C | C D | $+0.072\,p\,s$ | $-0.952\,w\,s$ |
| G, E′, C′ | D G | $+0.929\,p\,s$ | $+0.619\,w\,s$ |
| C | D G | $-0.072\,p\,s$ | $+0.619\,w\,s$ |
| G′, C′ | E F | $-0.500\,p\,s$ | $-0.381\,w\,s$ |
| C, E | E F | $+0.357\,p\,s$ | $-0.381\,w\,s$ |
| G′, C′ | F G′ | $+0.500\,p\,s$ | $+0.048\,w\,s$ |
| C, E | F G′ | $-0.357\,p\,s$ | $+0.048\,w\,s$ |
| C to C′ | A C | $+3.000\,p\,t$ | $+3.000\,w\,t$ |
| " | C E | $+3.857\,p\,t$ | $+3.952\,w\,t$ |
| " | E G | $+5.143\,p\,t$ | $+5.381\,w\,t$ |
| " | G G′ | $+5.857\,p\,t$ | $+5.952\,w\,t$ |
| " | B D | $-4.143\,p\,t$ | $-4.048\,w\,t$ |
| " | D F | $-5.857\,p\,t$ | $-5.619\,w\,t$ |
| " | F F′ | $-6.143\,p\,t$ | $-6.048\,w\,t$ |
| " | B C | $+1.857\,p$ | $+1.619\,w$ |

intersection of diagonals to lower chord, thus dividing the truss into double the number of panels, and modifications at the ends, it has been used as a pin-con-

nected structure for the Memphis and other bridges of very large span.

A somewhat similar method of subdividing the panels of a Pratt truss gives us the *Baltimore* truss. When the upper chord is inclined in addition, there results the *Petit* truss, which is the standard now for very long spans. The last two trusses named are really single intersection systems, with secondary trusses used to subdivide the panels. They thus admit of ready calculation without the approximations incident to double-intersection systems, as the Whipple and Lattice trusses given above. Larger treatises must be consulted for the computation of the stresses in such trusses.

### Analysis of Trusses for Wheel-Loads.

45. In the computation of the stresses in railroad bridges, the standard load is generally two consolidation engines, followed by a uniform car-load. In fig. 18 is shown the wheel diagram for Cooper's class E 40, taken from his specifications for 1896. The weights are those on one

rail, or for a single-track bridge, for a single truss.

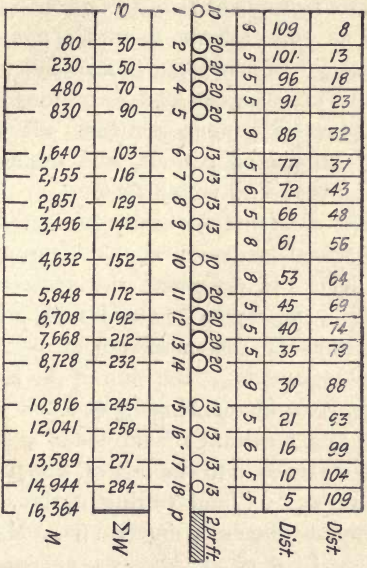


Fig. 18.

The figures above the wheels are, in order:

*The weights on wheels.*

*The distances apart of the wheels in feet.*

*The distances from p in feet.*

*The distances from wheel* 1 *in feet.*

The figures below the wheels are:

*The number of the wheel.*

*The sum of the loads from the left.*

*The sum of the moments about any wheel, of all wheel-loads on its left.*

The uniform car-load beginning at $p$ is 2 thousand pounds per foot; all weights are in thousand pounds, and the moments are in thousand-pound feet.

The moments are easily computed. Call the wheel-loads $w_1$, $w_2$, ..., beginning at the left, and designate the sum $(w_1 + w_2 + \ldots + w_n)$ by $w_{1-n}$; also let $M_n$ represent the sum of the moments of $w_{1-n}$ about wheel $n$ and $M_{n+1}$ the sum of the moments of $w_{1-(n+1)}$ about wheel $n + 1$.

If the resultant of the loads $w_{1-n}$ acts at a distance, $c$ to the left of wheel $n$, then $M_n = w_{1-n}\, c$, and calling $a$ the distance between wheels $n$ and $n + 1$, $M_{n+1} = w_{1-n}\,(c + a) = M_n + w_{1-n}\, a$. Thus since $M_2 = 10 \times 8 = 80$ and wheels 2 and 3 are 5 feet apart, $M_3 = M_2 + w_{1-2} \times 5 = 80 + (10 + 20)\ 5 = 230$; $M_4 = 230 + 50 \times 5 = 480$, and so on.

46. *Reaction for Wheel-loads.*—It is frequently necessary to find the left reaction R when only the wheel-loads ($w_1 + w_2 + \ldots + w_n$) are on the bridge; call the distance of wheel $n$ (which is nearest the right abutment) from the right abutment $e$; then the moment about that abutment of $w_{1-n}$ by the above $= w_{1-n}(c+e) = M_n + w_{1-n}e$; and if the span is $l$ feet,

$$R = \frac{M_n + w_{1-n}e}{l} \quad \ldots\ldots\ldots\ldots (18)$$

Thus, if for a span of 100 feet, wheel 14 is 5 ft. to left of right abutment, $R = (8728 + 232 \times 5) \div 100 = 75.68$, the quantities 8728 and 232 being found from the diagram under 14. Sometimes R is required when wheels 1 and 2 (or more) are off the span. In this case, as $M_n$ and $w_{1-n}$ include these loads, we must subtract the sum of their moments about wheel $n$ from $M_n$, and the sum of the loads off the span from $w_{1-n}$ before applying the formula.

Thus for the 100-feet span, let $w_{18}$ be 5 feet to left of right abutment; then wheels 1 and 2 are off the span. The sum of

their moments about wheel 18 $= 10 \times 104 + 20 \times 96 = 2{,}960$ and $w_1 + w_2 = 30$.

$$\therefore R = \frac{(14{,}944 - 2{,}960) + (284 - 30)\,5}{100},$$

or R = 132.54. We proceed similarly for any number of loads off the span.

47. To find the moment about a point $x$ feet to the right of $p$ of all loads on the left, including the uniform load 2 $x$, let the resultant of $w_{1-18}$ act $c$ feet to left of point $p$ of fig. 18, and call its moment about $p = M_p = w_{1-18}\ c = 16{,}364$; then the moment about the first point indicated is, $M_{p+x} = w_{1-18}\ (c + x) + 2\ x \times \frac{x}{2}$

$$\therefore M_{p+x} = 16{,}364 + 284\ x + x^2 \ldots . (19)$$

From this formula we find the following values:

$M_{p+5} = 17{,}809$, $M_{p+25} = 24{,}089$
$M_{p+10} = 19{,}304$, $M_{p+30} = 25{,}784$
$M_{p+15} = 20{,}849$, $M_{p+35} = 27{,}529$
$M_{p+20} = 22{,}444$, $M_{p+40} = 29{,}324$

48. *Reaction when car-load is on Bridge.* When the car-load extends a distance $x$ to left of right abutment the left reaction =

$M_{p+x} \div l$, if none of the loads 1, 2 etc., are off the span. In case wheels 1, 2 and 3 (say) are off the span, their moments about right abutment must be subtracted from $M_{p+x}$. Thus, for a 120-feet span and $25 = x$ feet of car-load on the bridge, the wheels 1, 2 and 3 will be found to be 134, 126 and 121 ft. respectively from the right abutment. The sum of their moments about it consequently $= 10 \times 134 + 20$ $(126 + 121) = 6280$; and this subtracted from $M_{p+25} = 24089$ gives 17809, the moment of the loads on the span about the right abutment. Dividing this by $l = 120$ gives the left reaction $R = 200.7$.

In examples of this kind, mark the distances along a straight line, and it will tend to avoid mistakes.

*Example I.*—For a span of 120 feet, $p$ is 15 feet to left of right abutment; compute R.

*Example II.*—For 120 feet-span, $p$ is 22 feet to left of right abutment; find R.

*Example III.*—With span = 60 feet, and right abutment 2 feet to right of wheel 14, find left reaction.

49. *Moment About an Apex on Loaded Chord.*—An example will sufficiently illustrate the method to follow in any case. Take a span of 100 feet, and suppose wheel 7 at an apex 40 ft. from left abutment; then $w_{16}$ is 4 ft. to left of right abutment.

The left reaction $= (12{,}041 + 258 \times 4) \div 100 = 130.73$ ∴ moment of R about apex $= 130.73 \times 40 = 5229.2$. The moment of loads $w_{1-7}$ about apex or wheel $7 = 2{,}155$ (from diagram). Therefore, the moment of reaction and loads left of apex, about apex as a centre of moments $= 5229.2 - 2155 = 3074.2$. In case any loads are off the span, R is found as explained in arts. 46 and 48, and the minus moment by subtracting from the tabular value the sum of the moments of the loads off the bridge about the apex considered.

*Example I.*—For a 78 feet-span find the moment at the centre when wheel 12 rests there.

*Example II.*—Do the same for a 100 feet-span.

50. *Moment Diagram.*—In fig. 19 is shown a moment diagram, constructed by

laying off the wheel-loads 1, 2, ..., in position to scale, drawing verticals through 1, 2, ..., to intersection with a horizontal base-line *v t*. From this base, the moment about any wheel of all loads to its left are taken from the diagram fig. 18, and laid off upwards to scale. Straight lines connect the points thus found, forming the moment diagram *v s f b r*.

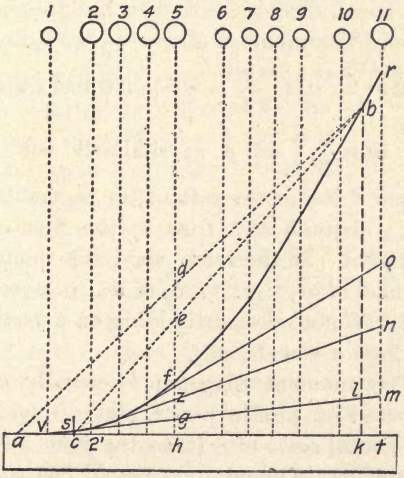


Fig. 19.

Another method of constructing the diagram is to compute the moment of each wheel 1, 2, ..., about some point as $t$ near the right end of the sheet and lay off $t$ m = moment of $w_1$ about $t$, $m$ $n$ = moment of $w_2$ about $t$, $n$ $o$ = moment of $w_3$ about $t$, and so on. Then draw $v$ $m$ to intersection with vertical through 2. From this point draw a line to $n$, and where it intersects a vertical through 3 draw a line to $o$, and so on. The moment of wheel-load 1, about $h$ (say) $= m\,t\,\frac{v\,h}{v\,t} = g\,h$; of wheel 2 about $h = m\,n\,\frac{2'\,h}{2'\,t} = g\,z$; similarly for the others. Hence the sum of the moments of $w_{1-4}$ about $h = h\,f$, as by the first construction. In the same way, the moment about $k$ of $w_{1-10} = k\,b$, of $w_{2-10}$ about $k = l\,b$ ($b$ not necessarily lying on a vertical through a wheel-load).

The moment diagram is usually constructed on profile paper, plate B with a horizontal scale of 8 ft. to the inch and a vertical scale about 1000 pound feet to 0.8 inch. Cross-section paper divided into

millimeters is very convenient, with a vertical scale about one half the above. One sheet will then contain the moments for about 150 feet length of train. The moments for trains including 40 feet of car-load have already been computed in Art. 47, and may be laid off at the proper distances to form the parabolic portion of the moment curve or equilibrium polygon.

To use this diagram to find reactions and moments, lay off, on a strip of the same paper used in constructing the diagram, the span and apices where moments are to be found. Then, if wheel 5 say, is to be placed at the apex $h$, slip the strip along until $h$ lies on the vertical through wheel 5 and the span $a\ k$ coincides with the base.

The vertical through $k$ intersects the equilibrium polygon at $b$ (fig. 19); draw $a\ b$ or place a string from $a$ to $b$; then noting the points $d$ and $f$ where $a\ b$ and the polygon intersect the vertical through 5, measure $d\ f$ to vertical scale, and its value is equal to the moment of reaction and loads to left of $h$ about $h$. The proof is easy; the sum of the moments of all

wheel-loads to left of $k$ about $k = k\,b$ (to vertical scale), the left reaction R at $a = k\,b \div a\,k$, and the moment of R about $h$ equals

$$k\,b\,\frac{a\,h}{a\,k} = h\,d.$$

The moment about $h$ of loads 1, 2, 3, 4 $= h\,f$, as shown above; therefore the moment of reaction and loads 1, 2, 3, 4 about apex $h$ of truss $= h\,d - h\,f = f\,d$, measured to the vertical scale.

If the span was $c\,k$, then load 1 is off the truss, and the moment of the left reaction at $c$ about $h = b\,l \times \frac{c\,h}{c\,k} = e\,g$; whence the moment about $h$ of reaction and loads 2, 3, 4 $= e\,g - f\,g = e\,f$.

The point $e$ is found by first drawing a vertical through $c$ to intersection $s$ with the equilibrium polygon; then $e$ is the intersection of $s\,b$ with the vertical through 5. Similarly we proceed if other loads are off the truss. The lines $a\,b$ and $s\,b$ are called *closing lines*. It is of course more accurate to measure the ordinates $k\,b$ or $l\,b$ to scale and to compute the moments

$d\,h$ or $e\,g$ by multiplying by the ratios $a\,h \div a\,k$ or $c\,h \div c\,k$; then the moments to be subtracted $h\,f$ or $f\,g$ can be taken directly from the table (fig. 18), only subtracting in the last case $w_1 \times v\,h$ from the tabular moment to find $f\,g$.

*Example I.*—For a span $a\,k$, suppose $k\,b$ to scale, reads 5240 and that $a\,h \div a\,k = \frac{4}{9}$; then $d\,h = \frac{4}{9}$ (5240) = 2328.9; $\therefore\ d\,f = d\,h - f\,h$ = 2328.9 − 830 (from table) = 1498.9 = moment about $h$ of reaction at $a$ and loads 1, 2, 3 and 4.

*Example II.*—Construct an equilibrium polygon on profile or cross-section paper to scales above for about 150 feet length of combined engine and car-load.

## Position of Live Load Giving Maximum Moment.

51. The following investigation applies either to a beam or to a single intersection truss with vertical members at each apex. In figure 20 let $w_n$ first rest at B and suppose that when all the loads are shifted to the left a distance $x_1$ the head of the uniform load ($p$ per foot) is at the right abutment C, so that no new load has

as yet come on the span and $w_{n+1}$ is $q$ feet to right of B.

Let $a$ = distance from $w_n$ to $w_{n+1}$ $\therefore$ $a = x_1 + q$.

$P = (w_1 + w_2 + \ldots + w_n)$ = sum of wheel-loads at and to left of B when $w_n$ is at B.

W = sum of all loads on span when $w_n$ is at B.

$d$ = panel length, $l$ = span, $b$ = A B, $y$ = a variable distance less than $q$.

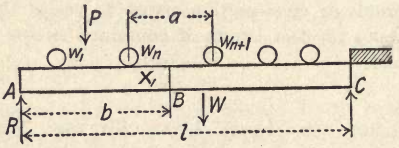


Fig.20.

Now, beginning with an initial position of the loads so that $w_n$ rests at B, on shifting all loads to the left a distance $x_1 + y$, the left reaction R is increased by

$$W \frac{x_1 + y}{l} + \frac{p y^2}{2 l},$$

and its moment about B by

$$b\left[\mathrm{W}\frac{x_1+y}{l}+\frac{p\,y^2}{2\,l}\right].$$

The minus moment of P about B is increased by P $(x_1 + y)$; hence the moment of R and P about B is increased by the shifting,

$$\frac{b}{l}\left[\mathrm{W}+\frac{p\,y^2}{2\,(x_1+y)}-\mathrm{P}\frac{l}{b}\right](x_1+y)\ldots(20)$$

If, in place of the uniform load, a new wheel-load Q just reaches C when the loads have been shifted a distance $x_1$, the left reaction is increased on shifting $y$ farther to the left, by $\mathrm{Q}\ y \div l$, and its moment about B by

$$\frac{\mathrm{Q}\,y}{l}\,b=\frac{b}{l}\left(\frac{\mathrm{Q}\,y}{x_1+y}\right)(x_1+y)$$

so that the term in (20) in the [ ] involving $p$ is replaced here by $\dfrac{\mathrm{Q}\,y}{x_1+y}$.

The following conclusions refer either to (20) or to the formula obtained from (20) by the last substitution.

We note in either case that

$$\frac{y}{x_1 + y} = \frac{1}{\frac{x_1}{y} + 1}$$ increases with $y$.

It is evident now from (20) that:

(1) So long as the [ ] is plus for $y = o$, it increases with $y$ and is a maximum for $y = q$, so that $w_{n+1}$ rests at B; when by the same method we ascertain whether the moment is increased on shifting loads still farther to the left, so that $w_{n+2}$ rests at B, and so on.

(2) It may happen that the [ ] is minus before new loads come on the span or for $y = o$; still if it is positive for some value of $y$, it is greatest for $y = q$; the moment is increased by the shifting, and $w_{n+1}$ is moved up to B.

(3) If, however, as new loads are added to those to the left of B, P is increased so much that the [ ] is minus, even for $y = q$ and $x_1 + y = a$, the moment is decreased by the shifting, and the weight last supposed at B before shifting must rest at B for maximum moment.

The case where some of the loads to the

left of B move off the span during the shifting will be considered in the example given farther on.

From (20), or the modified form when Q comes on the span, we have for the bracketed term, when $y = q$, $x_1 + y = a$,

$$\left[\mathrm{W} + m - \mathrm{P}\frac{l}{b}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

where,

$m = \left(\mathrm{Q}\frac{q}{a}\right)$ when the new load is a wheel-load and

$m = \left(\frac{p}{2}\frac{q^2}{a}\right)$ when the new load moving on the span is the uniform one. In the last case, when $x_1 = o$, $m = \frac{p}{2}a$, corresponding to the case where the uniform load is either on the span, or its front is at C, when $w_n$ is at B.

The criterion then for ascertaining whether a given position of the loads corresponds to a maximum moment at B is a simple one:

*Assume some wheel-load* $w_n$ *to rest at* B; *find* $\mathrm{P} = w_1 + w_2 + \ldots + w_n$ *and* W,

*the total load on the span when* $w_n$ *is at* B; *then if* (21) *above is plus, the moment is increased by moving* $w_{n+1}$ *up to* B; *if* (21) *is minus,* $w_n$ *rests at* B. *Hence if we assume at the start such a position of the loads that* (21) *is plus, shift loads to left. The instant* (21) *becomes minus, the last load* $w_n$ *at* B *before shifting must rest at* B *for maximum moment at that point.*

The above investigation for maximum moments and a following one for shears is taken from a paper by the writer on the subject in Van Nostrand's Magazine for March, 1885.

52. *Graphical Method.*—In fig. 21 the wheel-loads 1, 2, 3, ... are laid off to scale of distances, the proper horizontal distances apart and vertically to scale of loads. This can be done, to a large scale, on the moment diagram, fig. 19.

Mark off on a strip of paper the span A C of the beam or truss and any point, as B, where it is desired to find the position of loads giving maximum moment. To try wheel 3 at B, slip the strip along the base-line until B is vertically under 3.

Extend the vertical at C to meet the scale of loads at E.

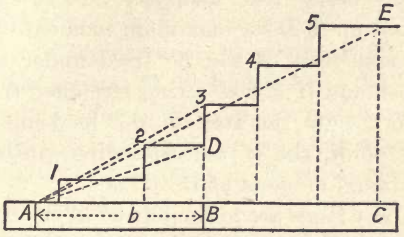


Fig. 21.

Neglecting $m$ in 21, we see that when $\left(\frac{W}{l} - \frac{P}{b}\right)$ is plus, the loads are shifted to left; when minus, they are not shifted. The [ ] changes sign, for a continuous loading, when the average load on the span is equal to that on A B. For the isolated loads, $\frac{W}{l} = \frac{C\,E}{A\,C} = tan$ C A E and $\frac{P}{b} = \frac{B\,3}{A\,B} = tan$ B A 3; hence as *tan* C A E < *tan* B A 3, $w_3$ is not moved farther to left. If we suppose $w_3$ slightly

to the right of B, $\frac{P}{b} = \frac{B\,D}{A\,B} = tan\ B\ A\ D$, which being less than *tan* C A E, $w_3$ is moved up to B for maximum moment. In any case then, B can be tried under any wheel, and if a fine string stretched from A to E cuts the step in the load-line at that point, the $w$ corresponding causes a maximum moment at B.

If any loads are to the left of A, the lines from E, D and 3 are drawn to a point where a vertical through A meets the load-line, as is evident.*

Unfortunately, this graphical method, as it neglects the influence of new loads moving on the span where the loads are shifted $a$ to the left, often requires that two or more contiguous wheels command at B, so that the corresponding moments have to be computed and compared, to see which position corresponds to maximum moment at B. For such cases the exact

* The graphical method outlined above was given by Dr. H. T. Eddy in a paper in the Transactions *Am. Soc. C. E.*, Vol XXII. 1890, pp. 259–358, where the whole subject for maximum moments and shears is discussed.

analytical method above (Art. 51) gives but one position, which is conclusive. An exact result though can be obtained by the graphical method, by laying off $m$ (page 125) to scale, vertically above E, fig. 21, to fix a point E′, so that C E′ = W + $m$ of (21). Then (W + $m$) ÷ $l$ = *tan* C A E′ and it follows, as above, that when A E′ cuts D 3, wheel 3 at B causes a maximum moment there.

It will now be assumed that the student will construct the load diagram (fig. 18) on cross-section paper to a large scale, so as to include about 150 feet of the load, writing the loads, moments, and distances on it, as in the figure; also, that he will construct a diagram of the truss next to be investigated to the same scale. On placing the first diagram below the second, so that the assumed wheel rests at the apex to be considered, the loads P, W, Q and the distances $q$, $a$ and $b$, are at once read off, the position of the load giving maximum moment for the apex quickly found and the maximum moment computed. It should then be checked

by use of the equilibrium polygon (fig. 19), as explained in article 50.

53. *Computation of the maximum stresses in the chords of a Pratt truss for wheel-loads.*—Let the span of the Pratt truss (fig. 22) be 120 feet, divided into 6 panels of 20 feet each, the height of truss 26 feet.

As we have seen above, a wheel-load always rests at an apex for maximum moment.

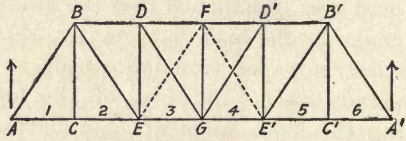


Fig. 22.

*Moment at C.*—Try wheel 2 at C.

On placing the load diagram along the truss diagram, we see that the front of the car-load is 1 foot to right of A', $P = w_1 + w_2 = 30$, $l \div b = 6$, $W = 284$, $a$ = distance between $w_2$ and $w_3 = 5$ and $q = 4$. Then, since for the car-load assumed $p \div 2 = 1$, we have (21) Art. 51,

$$\left[W + \frac{p\,q^2}{2\,a} - P\frac{l}{b}\right] =$$

$$284 + \frac{4^2}{5} - 30 \times 6 > 0,$$

hence $w_3$ is moved up to C.

Next, assuming $w_3$ at C, the car-load is on the bridge, $P = w_{1-3} = 50$, $W = 284 + 4 \times 2 = 292$, $a = 5 = q$, and

$$\left[W + \frac{p}{2} a - P \frac{l}{b}\right] =$$

$$292 + 5 - 50 \times 6 < 0.$$

Therefore, by Art. 51 $w_3$ rests, or *commands*, at C for maximum moment. For this position of the live load, $x = 4$ in *eq.* 19, Art. 47, therefore the left reaction $R = (16{,}364 + 284 \times 4 + 4^2) \div 120 = 145.967$ and its moment about $C = R \times 20 = 2919\frac{1}{3}$. The minus moment of $w_1$ and $w_2$ about $C = 230$ by the table; hence the moment of reaction and loads to left of C about $C = 2919\frac{1}{3} - 230 = 2689\frac{1}{3}$.

*Note.*—As the heavy wheel-loads more frequently command an apex than the lighter ones, it might be inquired whether $w_{1\,2}$ or $w_{1\,3}$ at apex C might not give a greater moment than the equally heavy load $w_3$; but with $w_{1\,3}$ (say) at C, the uniform load is seen to replace the second locomotive and tender for the first position when

$w_3$ is at C, the left reaction is thus less, and therefore the moment about C is smaller. The average load per foot of wheels 6, 7, 8, 9, 10 is about 62 ÷ 32, or nearly the same as the uniform load 2 per foot; but for a girder of over 60 feet span, it may happen that the maximum moment at some point of the span is greatest for the second engine and tender followed by the car-load, since there is a space of 11 feet in front of $w_{11}$ with only a load 10 on it, which would be replaced by a uniform load of $2 \times 11 = 22$ in all. Both positions should then be tried.

*Moment About* E.—Try $w_5$ at E; then $w_{18}$ is 1 foot to right of A′. $P = w_{1-5} = 90$, $l \div b = 6 \div 2 = 3$, $W = w_{1-17} = 271$, ∴ (21), Art. 51, gives

$$\left[W + \frac{Q\,q}{a} - P\,\frac{l}{b}\right] =$$

$$\left[271 + \frac{Q\,q}{a} - 270\right] > 0;$$

hence $w_6$ is moved up to E. It is not necessary, in this case, as in the majority of cases, to compute $Q\,q \div a$ or the corresponding expression for uniform load.

With $w_6$ at E, 3 feet in length of car-load is on the bridge. P = 103, W =

$284 + 3 \times 2 = 290$, $a$ = distance from $w_6$ to $w_7 = 5$ ∴ (21) of Art. 52 gives

$$\left[\mathrm{W} + \frac{p\,a}{2} - \mathrm{P}\,\frac{l}{b}\right] =$$

$$290 + 5 - 103 \times 3 < 0;$$

hence $w_6$ commands at E for maximum moment.

The moment of the left reaction about E equals

$$\mathrm{M}_{p+\frac{1}{3}}\frac{2\,d}{6\,d} = \tfrac{1}{3}(16{,}364 + 284 \times 3 + 3^2) =$$

$5741\frac{2}{3}$.

The minus moment of $w_{1-6} = 1640$ from the table ∴ max. moment at E = $4101\frac{2}{3}$.

It sometimes happens, for certain spans and panel-lengths, that a heavy load of the second engine commands at E, as then the large minus moment of the first engine is replaced by the smaller moment of the first tender. Let us try $w_{11}$ at E, in which case $w_{1-5}$ are off the span, $\mathrm{W} + m - \mathrm{P}\,\frac{l}{b} = 264 + 5 - 82 \times 3 > 0$; hence $w_{12}$ is moved up to E. There is now 40 feet of uniform load on the bridge. $\mathrm{W} + m - \mathrm{P} \times 3 = 274 + 5 - 102 \times 3 < 0$ ∴ $w_{12}$ commands at E.

With $w_{12}$ at E, $w_5$ is 6 feet to left of A. The

moment of $w_{1-18}$ and 40 feet of car-load about $A' = M_{p+40} = 29{,}324$ (Art. 47). From this we must subtract the moment of $w_{1-5}$ about $A' = M_5 + w_{1-5} \times 126$ (see Art. 46) $= 830 + 90 \times 126 = 12{,}170$, $\therefore$ moment of loads on span about $A' = 29{,}324 - 12{,}170 = 17{,}154$. The moment of the left reaction R about E is therefore $\frac{17.154}{6\,d} \times 2\,d = 5718$.

The minus moment of $w_{6-11}$ about $w_{12}$ at E is easily found to be $M_{12} - (M_5 + w_{1-5} \times 46) = 6708 - (830 + 90 \times 46) = 1738$; hence the moment of R and $w_{6-11}$ about E is $5718 - 1738 = 3980$.

This is less than $4101\frac{2}{3}$, found above, so that $w_6$ at E gives the greatest moment.

*Moment at G.*—With $w_9$ at G, the car-load is 1 foot to right of A′; $a = 8$, $q = 7$, $P = w_{1-9} = 142$, $l \div b = 2$, $W = 284$, $\therefore$ (21), $284 + \frac{7^2}{8} - 142 \times 2 > 0$ and $w_{10}$ is moved up to G. Note here that, if the new load moving on was neglected, either $w_9$ or $w_{10}$ could command at G, as (21) would then be zero.

Next try $w_{10}$ at G and move loads 4 feet to left only, whence $w_1$ rests at A.

Including $w_{11}$, $P = 152$, $W = 284 + 14$

$= 298$, and in (20), Art. 51, $x_1 = 0$ (as uniform load is on the bridge) and $y = 4$, $\therefore$ (20) becomes $2\,[298 + 4 - 152 \times 2] < 0$, so that apparently $w_{10}$ must be moved back to G; but a farther shifting of loads to the left moves $w_1$ off the span and changes this conclusion.

Thus with $w_1$ at A and not including it, $P = 142$, $W = 274 + 2 \times 11 = 296$, $x_1 = o$ and $y = 4$ in (20) which moves $w_{11}$ to G. $\therefore$ the [ ] of (20) gives

$$[296 + 4 - 142 \times 2] > 0,$$

so that the moment is increased by shifting to the left from the position $w_1$ at A; but whether $w_{10}$ at G or $w_{11}$ at G gives the greater moment can be determined only by computing the moments. We shall, in addition, compute the moment at G for $w_{12}$, at G, in which case $w_1$ and $w_2$ are both off the span.

By the method explained in Art. 49, we find for

$W_{10}$ at G, moment at G $= 4569$,
$W_{11}$ at G, moment at G $= 4596.5$,
$W_{12}$ at G, moment at G $= 4569$,

so that the heavy load $W_{11}$ commands at G for maximum moment.

The graphical method of Art. 50 is here a great saving in time, as moments like $ef$ in fig. 19 can be compared by use of dividers. When the maximum is thus found graphically, it can be computed afterwards if desired. The computation of moments for $w_{11}$ at G is as follows: $w_1$ is 124 feet to left of A′ (fig. 21) and $x = 15$; therefore the moment of R about G is (Arts. 47 and 49),

$$(M_{p+15} - w_1 \times 124)\frac{3}{6}\frac{d}{d} =$$

$$\tfrac{1}{2}\,(20{,}849 - 1240) = 9804.5$$

The minus moment of $W_{2-11}$ about G or $w_{11}$ equals 5848 (from table fig. 18) minus ($w_1 \times$ distance from $w_1$ to $w_{11} = 10 \times 64 = 640$) $= 5208$ ∴ moment of reaction and $w_{2-11}$ about G $= 9804.5 - 5208 = 4596.5$.

54. The maximum chord stresses are now found by dividing the maximum moments above by 26, the height of truss centre to centre of chords.

| Piece | Loading | Moment | Stress |
|---|---|---|---|
| A C E | $w_3$ at C | 2689.333 | 103.436 |
| B D, E G | $w_6$ at E | 4101.667 | 157.756 |
| D F | $w_{11}$ at G | 4596.500 | 176.789 |

The stresses are in thousand pounds.

### Position of Live Load Giving Maximum Shear in a Beam.

55. In fig. 20, Art. 51, let $w_1$ rest at B; the shear just to left of B is equal to $R = W \frac{s}{l}$, where $s$ = distance of W (the resultant of total load on the bridge) from C when $w_1$ is at B. On shifting to left a distance $a$, when $w_2$ rests at B, the shear is

$$W \frac{s + a}{l} + Q \frac{q}{l} - w_1$$

$Q\,q \div l$ being the increase of R due to a new wheel-load Q coming on the span, where $q$ is its distance from C when $w_2$ is at B. The shear is increased when

$$W \frac{a}{l} + Q \frac{q}{l} - w_1 > 0;$$

so that $w_2$ rests at B. If $(W\ a + Q\ q - w_1\ l)$ is minus, $w_1$ rests at B for maximum shear. If $w_2$ is found to move up to B, call $W_1$ the resultant of all loads on the span when $w_2$ is at B, and $s_1$ its distance from C. Then if loads are again shifted to left a distance $a_1$, so that $w_3$ moves up to B, the shear is

$$W_1 \frac{s_1 + a_1}{l} + \frac{Q\ q}{l} - (w_1 + w_2)$$

and since the shear with $w_2$ at B is $W_1 \frac{s_1}{l} - w_1$, the difference is

$$W_1 \frac{a_1}{l} + \frac{Q\ q}{l} - w_2$$

when this is plus, $w_3$ rests at B for a greater shear; when minus, $w_2$ commands there.

For usual length of girders and usual loads, it is found that $w_1$ or $w_2$ (but not $w_3$) commands at points for maximum shear.

### Position of Live Load Giving Maximum Shears in any Single Intersection Truss.*

55 (*a*). In fig. 23 let

* For a graphical solution see larger treatises.

$P = (w_1 + w_2 + \ldots + w_n) =$ sum of wheel-loads at and to left of C when $w_n$ is at C.

W = total load on span when $w_n$ is at C, $a =$ distance from $w_n$ to $w_{n+1} = x_1 + q$, $d =$ panel-length, $l =$ span, N = total number of panels, $y =$ a variable distance less than $q$.

First suppose $w_n$ at C, and that when all the loads are shifted a distance $x_1$ to the left a new wheel-load Q rests at A′; then on shifting a farther distance $y$ to the left, the left reaction R is increased over the value corresponding to $w_n$ at C,

$$W \frac{x_1 + y}{l} + Q \frac{y}{l},$$

and the load held at B by $P \frac{x_1 + y}{d}$.

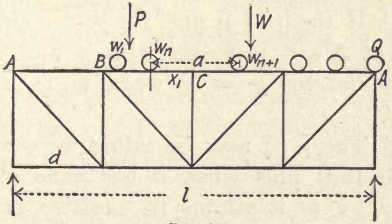


Fig. 23.

The shear in panel B C is thus altered by the difference,

$$\left[W + Q\frac{y}{x_1 + y} - P\frac{l}{d}\right]\frac{x_1 + y}{l} \quad .. (23)$$

If the uniform load just reached A′ when the loads were shifted $x_1$ to left, the term $Q\frac{y}{x_1 + y}$ would be replaced by $\frac{p\,y^2}{2\,(x_1 + y)}$. Prove this. Observe in (23) that $\frac{y}{x_1 + y} = \frac{1}{\frac{x_1}{y} + 1}$ increases with $y$.

By a consideration of the [ ] in (23), we see that the shear is increased by the shifting when it is plus, diminished when it is minus; or, more specifically:—

(1) If the [ ] is plus for $y = o$ (when Q is at A′), it increases with $y$ and is a maximum for $y = q$ when $w_{n+1}$ rests at C.

(2) The [ ] may be minus for $y = o$, but if it is plus when $y$ has some other value ($< q$) it attains its positive maximum for $y = q$ or $w_{n+1}$ at C.

(3) If, however, the [ ] is minus, even for $y = q$, the shear is diminished by any shifting; hence $w_n$ rests at C.

We reason similarly when the uniform load comes on; so that, in either case, noting that when $y = q$, $x_1 + y = a$, the expression to estimate is

$$[W + m - P N] \ldots\ldots\ldots\ldots (24)$$

where, $m = \left(Q \frac{q}{a}\right)$ for a new wheel-load

and, $m = \left(\frac{p}{2} \frac{q^2}{a}\right)$ for the new car-load,

which last reduces to $\left(\frac{p\,a}{2}\right)$ when $x_1 = o$.

The application is obvious. If $w_2$, say, is placed at an apex and (24) is plus, by case (1) above, the loads are shifted until $w_3$ reaches the apex. Then with $w_3$ at the apex, if (24) is minus the shear, by case (3) above, would be decreased by any shifting; hence the shear in the panel just left of the apex is a maximum for $w_3$ at the apex. Similarly for other cases. It generally happens in practice that wheels 1, 2 or 3 command at apices. Some wheel, as,

we have seen above, is always at an apex for maximum shear.

If the panels are so short that $w_1$, or both $w_1$ and $w_2$, pass to the left of B during the shifting, it is best to compute the shears for the different positions and compare them directly.

## Position of Live Load for Maximum Shears in a Pratt Truss.

56. Consider again the Pratt truss, fig. 22, Art. 53, of 120 feet span, divided into 6 panels of 20 feet each $\therefore$ N = 6 in (24), Art. 55, which becomes

$$[W + m - 6\ P] \ldots\ldots\ldots\ldots (25)$$

There is no need of considering $m$ when $W - 6\ P > 0$; but when $W - 6\ P < 0$, $m$ may have such a value as to make $W + m - 6\ P > 0$, in which case it should be computed. When $q =$ its maximum value $a$, $m = Q\ q \div a$ cannot exceed 20 the largest value of Q.

*Apex* C′.—Try $w_1$ at C′ $\therefore$ P = 10, W = 70 and (25), $[70 + m - 60] > 0$; hence $w_2$ is moved up to this apex. If we con-

sider in the same way other apices to the left, $w_2$ is moved up, as W in (25) is larger than 70 and 6 P = 60 remains the same; hence $w_1$ need not be tried at the other apices. With $w_2$ at C′ (25) becomes [90 + $m$ — 180]. As $m$ < 20 always, 90 + $m$ — 180 < 0; hence the loads are not shifted farther to the left, and $w_2$ commands at C′.

*Apex* E′.—With $w_2$ at E′, 48 feet of load is on the span and $w_9$ is at A′. ∴ Q = 13, $q = a = 5$, W = 129, P = $w_1 + w_2 = 30$ and from (25), [129 + 13 — 180] < 0, so that $w_2$ commands at E′.

*Apex* G.—With $w_2$ at G, 68 feet of load is on the bridge, ∴ $w_{12}$ = Q is one foot to right of A′; hence $q$ = 4, since $a$ = distance between $w_2$ and $w_3$ = 5. Also W = 172, P = 30 and from (25), [172 + $\frac{4}{5}$ 20 —180] > 0; hence $w_3$ at G will give a greater moment than $w_2$, and the same happens for all apices to the left, since W is greater than for this case. Here, the influence of the new load moving on is seen to cause $w_3$ to move up to G, whereas if it were neglected $w_2$ would command the apex.

*Apex* E.—with $w_3$ at E, 93 feet of live load is on bridge, W = 245, P = 50, ∴ from 25, $[245 + m - 300] < 0$; hence $w_3$ commands the apex and, *a fortiori*, it commands at G.

*Apex* C.—Wheel 3 at *c*, gives W = 284 + 4 × 2 = 292, P = 50, $a = 5 = q$, and as the uniform load is on the bridge, $m = \frac{p\,a}{2} = a = 5$; hence (25),

$$[292 + 5 - 300] < 0;$$

hence $w_3$ again commands the apex. The position for maximum shear at C is the same as for maximum moment, Art. 53, so that if the first computation has been made it is not necessary to make the second.

It may be interesting to note that if there were $(6 + e)$ panels of 20 feet each in place of 6, $w_3$ would still command at the apex next the left abutment, since W is increased by $40\,e$ only and P N by $50\,e$ over the last values.

57. *Maximum Shears and Stresses in Pratt Truss.*—The position of the live load giving maximum shear in any panel having

been found as above, the shear is equal to the left reaction minus the part of the live load on the panel held at its left apex, which last is equal to the moment of the load in the panel about the wheel at its right end divided by the panel-length. This moment is taken from the table, fig. 18. The method of finding the reaction has been given in articles 46 and 48. The student should mark off the distances required (by aid of fig. 18) along a straight line, or use the diagrams mentioned, as in no other way can the results below be made so plain.

*Max. Shear in* A B, fig. 22.—With $w_3$ at C, the reaction was found above to be 145.967. The sum of the moments of $w_1$ and $w_2$ about $w_3$ (fig. 18) is 230, and this divided by 20 gives the part of the load on A C held at A = 11.5, ∴ shear in A B = 145.967 — 11.5 = 134.467.

*Max. Shear in* B E. $w_3$ is at E; $w_{16}$ is at A′; shear = 12,041 ÷ 120 — 11.5 = 88.843.

*Max. Shear in* G D and D E. $w_3$ at G; $w_{12}$ is 4 feet to left of A′.

R = (6708 + 192 × 4) ÷ 120 = 62.3,
∴ shear = 62.3 — 11.5 = 50.8.

*Max. Shear in E′ F and F G.* $w_2$ is at E′ and $w_9$ at A′.

Shear = (3496 ÷ 120) — (80 ÷ 20) = 25.133.

*Shear in E′ B′* with $w_2$ at C′.—We find $w_5$ to be 5 feet to left of A′, ∴ R = (830 + 90 × 5) ÷ 120 = 10,667. As the part of $w_1$ held at E′ = 80 ÷ 20 = 4, the shear is 6.667.

This shear alone would cause compression in E′ B′. Similarly, with $w_2$ at C and load extending to left abutment, the compression in B E = 6.667 × *sec* $i$ = 8.414, as noted below; and this, combined with the dead-load tension in B E, gives its minimum stress.

We have here, *tan* $i$ = 20 ÷ 26 = .76923, ∴ $i$ = 37° 34′ and *sec* $i$ = 1.262.

On multiplying shears on end post, ties and counters by 1.262, their stresses are found and entered below.

The dead-panel load for this truss (see Art. 5, *eq.* 5) is 13.5, one-third of which is assumed on the upper chord. The stresses

(tension = +, compression —)are found as in Art. 38, and then combined with the live-load stresses.

MAXIMUM AND MINIMUM STRESSES IN END POST AND DIAGONALS.

| Load | A B | B E | D G | E′ F |
|---|---|---|---|---|
| Live | — 169.967 | + 112.120<br>— 8.414 | + 64.110 | + 31.718 |
| Dead | — 42.854 | + 25.555 | + 8.520 | — 8.520 |
| Max. | — 212.821 | + 137.675 | + 72.630 | + 23.198 |
| Min. | — 42.854 | + 17.141 | 0 | 0 |

MAXIMUM AND MINIMUM STRESSES IN VERTICALS.

| Load | D E | F G | B C |
|---|---|---|---|
| Live | — 50.800 | — 25.133 | + 65.550 |
| Dead | — 11.250 | + 2.250 | + 9.000 |
| Max. | — 62.050 | — 22.883 | + 74.550 |
| Min. | — 4.500 | — 4.500 | + 9.000 |

As the max. and min. stresses in B E are both tension, no counter is required in panel C E. The minimum stress in D G

$= o$ when E F is in action and stress in E′ F $= o$ when G D′ is in action.

The dead-load stress, 2.25 tension, in F G occurs when F E′ is in action, and is the shear in F G due to dead load alone. It is the part of the dead load 9 at G that must go to the right abutment to make both reactions, at A and A′, equal; and this part must travel up F G, since G D′ is supposed out of action. The maximum stress in F G can likewise be found as in Art. 38. Thus shear in F E′ due to live and dead loads $= 25.133 - 6.750 = 18.383$. The stress in F G $= 18.383 + 4.5$ (dead load at F) $= 22.883$, as given above.

The minimum stresses in D E and F G, given above, are those due to dead loads at top of posts when the diagonals connecting with their tops are out of action.

The (minimum) dead-load stress in the hip vertical B C is simply the dead load held at C; the maximum stress is determined in the next article.

The minimum stresses in the chords are those caused by dead load.

MAXIMUM AND MINIMUM STRESSES IN CHORDS. (SEE ART. 54).

| Load | A C E | B D, E G | D F |
|---|---|---|---|
| Live | 103.436 | 157.756 | 176.789 |
| Dead | 25.960 | 41.536 | 46.728 |
| Max. | 129.396 | 199.292 | 223.517 |

58. *Maximum Live-load Stress in Hip Vertical.*—In fig. 24, two end panels are shown. As in skew bridges the panel-lengths are often unequal. Let $12 = d_1$, $23 = d_2$. Also, suppose $w_n$ to rest at 2 and P = resultant of $w_{1-n}$ to act a distance $c$ to left of 2 and W the resultant of all the wheel-loads on 1 2 3 to act a distance $s$ to left of 3. Then calling $R_1$ the reaction of the floor beam at 1 or part of the load on 12 supported at joint 1 and $R_2$ the part of the load on 123 held up by the hip vertical 2 B, we have, taking moments about joints 2 and 3 of the reactions and loads on the left, $R_1 d_1 = P c$; $R_1 (d_1 + d_2) + R_2 d_2 = W s$. Substituting in the last equation the value of $R_1$ obtained from the first,

$$R_2 = \left[ W s - P \frac{d_1 + d_2}{d_1} c \right] \frac{1}{d_2}$$

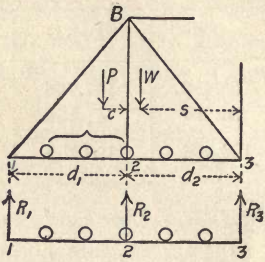


Fig. 24.

As before, let $a$ = distance from $w_n$ to $w_{n+1}$, and suppose loads shifted to left a distance $x < a$ when a new wheel-load Q is supposed to move on the panel 2 3 a distance $y$, then $s$ and $c$ are each increased by $x$ and *the change* in $R_2$ is

$$\left[ W x - P \frac{d_1 + d_2}{d_1} x \right] \frac{1}{d_2} + Q \frac{y}{d_2} \quad \ldots (26)$$

If this is plus for some value of $x$, the increase of $R_2$ is greatest for $x = a$ when $y = q$ (say), so that $w_{n+1}$ rests at 2. Again, determining P, W, Q for $w_{n+1}$ at 2, if now (26) is minus even for $x = a$, $y = q$,

$R_2$ is diminished by any farther shifting, so that $w_{n+1}$ rests at 2 for maximum $R_2$.

Making $x = a$, $y = q$ in (26), we have,

$$\left[W + Q\,\frac{q}{a} - P\,\frac{d_1 + d_2}{d_1}\right]\frac{a}{d_2} \quad \ldots\ldots(27)$$

When the [ ] is plus, we shift to left, and continue thus until it changes to minus. The instant this occurs, the last load supposed at 2 remains there for maximum $R_2$.

By reference to Art. 51, this is seen to be the criterion for finding what wheel-load should command at 2 for a maximum moment there when the span of the bridge is $13 = d_1 + d_2$.

We have supposed above that no loads on 1 2 pass to the left of 1; if any do, it is best to compute $R_2$ directly for two or more positions of the live load, and thus determine maximum $R_2$ by trial.

The value of $R_2$ above can be put in a form very convenient for computation. Call $M_2$ and $M_3$ the sum of the moments of the wheel-loads to left of 2 and 3 about 2 and 3 respectively; then $P\,c = M_2$, $W\,s = M_3$,

$$\therefore R_2 = \left[M_3 - \frac{d_1 + d_2}{d_1} M_2\right] \frac{1}{d_2} \quad \ldots\ldots (28)$$

To apply the above theory to finding the maximum live-load stress ever sustained by the hip vertical in the Pratt truss having panels of 20 feet each, the [ ] in (27) becomes, since $d_1 = d_2$,

$$\left[W + Q \frac{q}{a} - 2P\right] \ldots\ldots (29)$$

Suppose $w_3$ at 2, then $W = w_{1-6} = 103, P = 50, \therefore W - 2P > 0$ and $w_4$ at 2 gives a greater $R_2$. For this position of the loads, $W = w_{1-7} = 116, P = 70$ and no new load moves on when $w_4$ is shifted 5 feet to left. $[W - 2P] < 0, \therefore w_4$ rests at joint 2 for maximum $R_2$.

$$\text{From (28), } R_2 = \frac{M_3 - 2M_2}{d} \quad \ldots\ldots (30)$$

By use of table, fig. 18, $M_3 = 2155 + 116 \times 1 = 2271$, also $2M_2 = 2 \times 480 = 960$ and $d = 20, \therefore R_2 = 65.55$, as given in the preceding article.

When new loads moving on are neglected, we note that (29) shows that, for

equal panel-lengths the loads on the two panels, if continuous, should be equal.

If there are two lines of stringers, as is usual, the maximum moment and shear for a floor beam correspond to two loads of 65,550 pounds, each applied at the points where the stringers are riveted to the floor beam.

59. *Maximum Moment in Stringer.* Let P = resultant of loads at B and to its left, fig. 25, and $c$ its distance from B, W = resultant of all wheel-loads on stringer A D and $g$ its distance from wheel at B. Then as the distance A B = $x$ varies, the loads shifting at the same time, so that the wheel-load first supposed at B remains always at B, the moment at B varies and must have a maximum for some value of $x$.

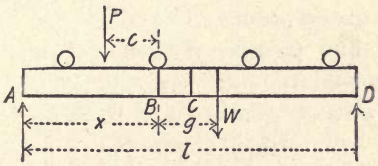


Fig.25.

This moment at B is

$$M = \frac{W}{l}(l - x - g)\,x - P\,c \quad \ldots\ldots(31)$$

For a maximum, its derivative with respect to $x$ must be zero.

$$\therefore (l - 2x - g) = o, \text{ or, } x + \frac{g}{2} = \frac{l}{2}$$

Therefore the maximum moment under the particular wheel at B occurs when the load at B is as far from the centre C of the beam on one side as the centre of gravity of the loads on the beam is on the other side of the centre.

Article 52 shows that wheels 3 or 4 command at C for maximum moment there, for $l = 20$, when only the heaviest wheel loads 2, 3, 4 and 5 are on the stringer, hence, wheel 3 will be tried at B for the greatest of all possible moments at any point of the beam.

To find $g$, take moments of $w_2$, $w_3$, $w_4$, $w_5$ and W, about $w_3$

$$W\,g = w_4 \times 5 + w_5 \times 10 - w_2 \times 5$$

$$\therefore g = \frac{20\,(5 + 10 - 5)}{80} = 2.5.$$

Hence, $w_3$ is placed 1.25 feet to left of C, in which case W would be 1.25 feet to right of C.

The moment about B is then easily found to be 206.25. The same result follows if the loads are reversed, and $w_4$ is taken at B; hence, this is the greatest moment ever experienced at any point of the stringer as the rolling load moves over it in either direction. The maximum moment is most readily found by substituting the value of $x = \frac{1}{2}(l - g)$ corresponding to it, in (31) above, giving

$$\text{max. M} = \frac{\text{W}}{4\,l}(l - g)^2 - \text{P}\,c \ldots\ldots\ldots (32);$$

or for this case

$$\text{max. M} = \frac{80}{4 \times 20}(17.5)^2 - 20 \times 5 =$$

206.25 as before.

When $w_3$ is at C, the moment there is 200, or less than the above.

The maximum shear in the stringer occurs when $w_2$ is at A, whence $w_5$ is 5 feet from D. The shear then is equal to the left reaction, and is

$$\frac{20}{20}(5 + 10 + 15 + 20) = 50.$$

*Example.*—Find the maximum moment in a floor joist, 20 feet long, sustaining two road roller loads of 3200 and 2880 pounds respectively, 11 feet apart.

60. *Position of Load for Maximum Moment in a Warren Girder.*—In Art. 51 the method was given for finding the position of live load corresponding to maximum moment at any point of a beam, the Pratt or Howe truss, or the Warren with sub-verticals, fig. 5. The same method evidently applies to the Warren proper, when the centre of moments is taken on the chord that bears the roadway, but not when the centre is taken on the other chord. The strict investigation is then briefly as follows:

In fig. 26, let the roadway be on the lower chord, and suppose it is desired to find the loading that will cause a maximum stress in D E. B is the centre of moments. Call the horizontal distances from A and D to B, $b$ and $c$ respectively;

also let $W_1$ = resultant of loads on A D, including any load at D, $W_2$ = resultant of loads on D E, including any load at E and $W_3$ = resultant of loads on rest of span, E A′. Let $W = W_1 + W_2 + W_3$ and suppose no load to move off, but a new load Q to move on the span, a distance $y$, when all the loads are shifted to the left a distance $x$, in which case R is increased by

$$W \frac{x}{l} + Q \frac{y}{l},$$

and its moment about B by

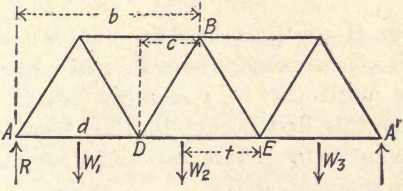


Fig. 26.

$$b \left( W \frac{x}{l} + Q \frac{y}{l} \right).$$

The minus moment of $W_1$ about B is increased by $W_1 x$.

The amount of $W_2$ transferred by the stringers to D is $W_2 t \div d$; this is increased

by the shifting (if no loads pass D or E), $W_2\ x \div d$ and its moment about B by $W_2\ c\ x \div d$. The total moment about B is thus altered an amount equal to

$$b\left(W\frac{x}{l} + Q\frac{y}{l}\right) - W_1 x - W_2\frac{c\ x}{d}.$$

If this expression is plus for $x = e$, $y = q$, when a load reaches D or E, the moment is increased by the shifting; if minus, it is decreased. This expression, for $x = e$, $y = q$, can be put in the form,

$$\left[W + Q\frac{q}{e} - \left(W_1 + W_2\frac{c}{d}\right)\frac{l}{b}\right]\frac{b\ e}{l};$$

hence if the [ ] here is plus, we shift until a load reaches D or E, and continue thus until the [ ] changes to minus. When this first occurs, the last load supposed at D or E remains there for maximum moment at B.

When a uniform load $p$ per foot moves on the span a distance $q$ when the loads are shifted to left $x = e$, the term $Q\ q \div e$ above is replaced by $\left(\frac{p}{2}\frac{q^2}{e}\right)$, which becomes $\left(\frac{p}{2}\ e\right)$, when $q = e$.

On comparing the above bracketed term with (21) Art. 51, we note that $P\frac{l}{b}$ of (21) is replaced by $\left(W_1 + W_2\frac{c}{d}\right)\frac{l}{b}$, which becomes identical with the former, for $c = o$, as should be the case.

For the Warren girder, $c \div d = 0.5$.

The above investigation applies also when the upper chord is inclined as in the Bowstring, Pegram and other trusses, as the shape of the unloaded chord does not enter into the investigation.

*Example I.*—Find the position of the loads (fig. 18) that causes a maximum moment in the third loaded chord-panel of a through Warren girder (fig. 1) of 108 feet span divided into 6 panels.

*Example II.*—For the same system of loads, find the maximum live-load stresses in a Pratt deck bridge having 5 panels, each 22 feet long, the height of truss being 24 feet, giving the position of the loads for each case.

61. *Equivalent Loads.*—The method of finding maximum stresses for the actual wheel-loads, proving very laborious in practice, attempts have been made to substitute

a so-called "equivalent" uniform load, either alone or in connection with one or two locomotive excesses. Such loads are found as will give the same stresses in certain members as the actual wheel-loads and these loads are then assumed for all the other members of the bridge. If one locomotive excess is used, it is placed at the head of the uniform load for maximum shears; if two excesses are used, the second is placed about 50 feet or more back of the first. For maximum chord stresses, the uniform load covers the span, and one locomotive excess is placed, in addition, at the apex about which moments are taken.

As to the agreement by the two methods, the reader may consult the elaborate papers by J. A. L. Waddell in Transactions of the American Society of Civil Engineers; Vol. XXVI., p. 71, and Vol. XXXI., p. 213, also Johnson's *Framed Structures*, chap. VI., where the possibilities and drawbacks of the approximate methods are fully discussed.

### Wind Pressure.

62. The wind is assumed to blow at right angles to the bridge, and to exert a maximum pressure of 30 pounds per square foot against the sides of both trusses, the entire flooring system (including track and hand-railing if any) and a train of cars also in case of a railroad bridge. A force of 30 pounds per square foot, and even less, will overturn an empty freight car, and thus wreck a bridge loaded with cars, which accounts for the limit imposed. For highway bridges this limit is sometimes exceeded, though the pressure above, corresponding to a hurricane, is rarely experienced in the life-time of any metal bridge.

It is customary to conceive a plane midway between the upper and lower chords of a bridge and to regard the upper lateral system as carrying all the wind pressure on the parts above the plane and the lower lateral system all the wind pressure on the parts below this plane.

The total area exposed to the pressure above is equal to the sum of the vertical

projections on a plane parallel to a truss or perpendicular to the direction of the wind of all parts of the bridge, including both trusses.

To avoid this computation, a common specification now is to allow 150 pounds pressure for each foot of span to both the upper and lower horizontal lateral trusses, to be treated as dead load ; also on highway bridges, 150 pounds per foot of span on the loaded chord to be treated as moving load and for railroad bridges 300 pounds, similarly treated. This last corresponds to 30 pounds pressure on a car body 10 feet in height. The dead load above is sometimes increased 0.4 pounds per lineal foot for each additional foot in length of span over 200 feet, 150 pounds being allowed up to 200 feet span.

Let us use this approximate rule for the Pratt truss, fig. 27 (*a*) of 6 panels of 20 feet each, the depth of truss being 26 feet. Figs. *b* and *c* are the horizontal projections of the upper and lower horizontal bracing between similar chords of the two trusses.

63. *Lateral Trusses.*—For the lateral

truss between the two upper chords we have by the rule above, the panel-loads 20 $\times 150 = 3000$ pounds or 1500 pounds at each panel-point $b$ to $b'$ and B to B′, the load being supposed divided between the two chords, as the wind pressure on both

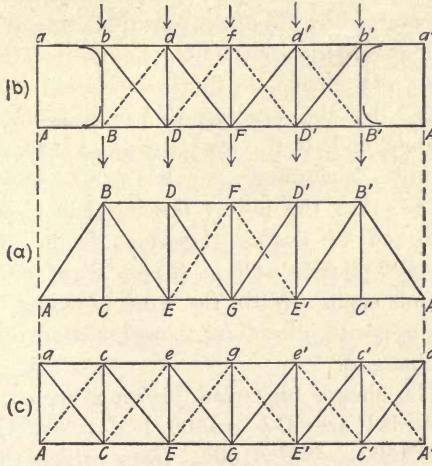


Fig. 27.

trusses is included. The pressures at B, B′, $b$, $b'$ are each about 1500 as at the other

apices, since the pressure on half the batter braces (end posts) is carried there. If this wind load is supposed to act from $f$ towards F, fig. ($b$), the full diagonals act; if in the other direction, the dotted diagonals. For the first case, the loads are carried by the lateral trusses to B, $b$, B′, $b'$, and there transferred by aid of the portal bracing and end posts to the abutments. Leaving off the loads at the ends, the total wind load transferred by the bracing to $b$ or $b'$ is 4500 pounds, or half the load acting at interior points. Including now that at the ends, we see that the portal bracing has to be designed to sustain loads at $b$ and $b'$ of 6000 pounds, and at B and B′ of 1500 pounds each. With the wind blowing in the opposite direction, these loads are interchanged.

The shears for wind blowing from $f$ towards F are: D $b$ = 4500, D $d$ = 3000, F $d$ = 1500, $f$ F=1500, from which the stresses in web members and chords are found for a distance between trusses of 16 feet, centre to centre, as follows:

D $b = +5625$, D $d = -3000$,
F $d = +2400$, $f$ F $= -1500$,
B $d = o$, D F $= +5625$,
$b\ d = -5625$, $d\ f = -7500$.

The chord on the windward side is thus in compression, the leeward chord in tension.

For *the lower lateral truss* (*c*) we have, as before, 1500 pounds fixed load at each intermediate apex, and for a railroad bridge 3000 pounds moving apex load. Both are supposed to be applied at the joints of both chords. Wind is supposed to blow in direction $g$ G.

The dead-load stresses are :

$a$ C $= +12{,}000$, $c$ E $= +7200$, $e$ G $= +2400$.

$c$ C $= -6000$, $e$ E $= -3000$, $g$ G $= -1500$.

A $c = 0$, C E $= +9375$, E G $= +15{,}000$.

$a\ c = -9375$, $c\ e = -15\,000$, $e\ g = -16{,}875$.

For full live-load only the chord stresses are double the above. For maximum shears due to moving load, (14) of Art. 32 gives

the shears on the diagonals, $S = (6 - n)(7 - n) \times 500$; therefrom the maximum stresses due to moving-load in ties and posts are as follows:

$a\,C = +\,24{,}000$, $c\,E = +\,16{,}000$, $e\,G = +\,9600$.

$c\,C = -\,13{,}000$, $e\,E = -\,9000$, $g\,G = -\,6000$.

For live and dead-loads combined, $g$ G bears a stress $= -\,6000$, as then counter $g\,E'$ is in action, and dead-load shear in $g$ $G = 0$. The stresses in thousand pounds are then as follows:

| | $a$ C | $c$ E | $e$ G | $c$ C | $e$ E | $g$ G |
|---|---|---|---|---|---|---|
| Wind on truss. | +12.0 | + 7.2 | +2.4 | — 6.0 | —3.0 | 0.0 |
| " " train. | +24.0 | +16.0 | +9.6 | —13.0 | —9.0 | —6.0 |

The chord stresses above are entered in the table, Art. 66, opposite "wind on truss" and "wind on train" for both directions of the wind.

64. *Overturning Action of Wind on Truss.*—In fig. 28 is shown a cross-section of the bridge and of a car on the rails.

The total wind force on the windward upper chord is $5 \times 1500 = 7500$ pounds, and the same force acts on the leeward chord as marked on the figure. These forces tend to overturn the bridge, the axes of rotation being the roller or other supports on the leeward side. This axis is a little below the lower chord; but, as the wind really acts a little below the top chord, call the arm 26 feet, the depth of truss. If we call $x$ the increased pressure

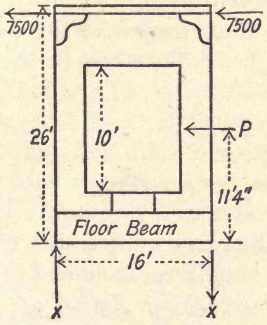


Fig.28.

at the two leeward supports, this same amount will represent the decrease of

pressure at the windward supports, for taking moments about either support we have $15{,}000 \times 26 = 16\,x$

$$\therefore \frac{x}{2} = 12{,}187$$

gives the vertical pressure at one leeward support or the decrease of pressure at one windward support. This vertical force then acts as a reaction on the end post A B, fig. 27 (*a*), and causes a stress in it $= 12{,}187 \times 1.26 = 15{,}356$, where 1.26 is the secant of the angle it makes with the vertical. The tangent of the same angle $= 0.769$; hence the stress in the chord A A′ equals $12{,}187 \times .769 = 9371$.

On the leeward truss, this gives compression in end post, and tension in the lower chord; in the windward truss these stresses are reversed. It is seen that this increase or decrease of pressure in chords is uniform from end to end.

65. *Overturning Action of Wind on Train.*—Let us suppose a car body 10 feet high, on which the wind exerts a pressure of 30 pounds per square foot. For a panel-length, or 20 feet length of car, the force

of the wind is $P = 10 \times 20 \times 30 = 6000$. The centre of pressure on a freight car is about 7 feet above the rails; call it 11 feet 4 inches above the centre of the lower chord. If $x'$ denotes the increase of pressure per panel on the leeward truss, and a like decrease of load on the windward truss, we have

$$16\, x' = 6000 \times 11\tfrac{1}{3}$$
$$\therefore x' = 4250.$$

Regarding this load per panel as a moving-load, we find the maximum stresses due to it in the leeward vertical truss (Art. 32); A B = — 13,380, B E = + 8920, D E = — 4250, D G = + 5355, F G = — 2125, F E′ = + 2677, E′ B′ = — 892.

For the panel-load $x' = 4250$, at each lower apex we find the chord stresses in the vertical truss.

A C E = + 8175, E G = + 13,080, B D = — 13,080, D F = — 14,715.

For the wind blowing in the opposite direction, the signs of all these stresses are changed.

66. These stresses are entered in the following tables under the head "wind

overturning." We may conceive the bridge to lie north and south, A being the north end, so that when the wind blows from the *east*, A B B′ A′ is the leeward truss, and when from the west the windward truss. The live and dead-load stresses found in Art. 57 for the same truss are given first in the tables, and then combined with the stresses due to wind, in order to get the total maximum and minimum corresponding to the most adverse conditions simultaneously occurring.

| | Main Ties | | Counter | Verticals | | |
|---|---|---|---|---|---|---|
| | B E | D G | F E′ | B C | D E | F G |
| Dead load | + 25.6 | + 8.5 | — 8.5 | + 9.0 | —11.3 | + 2.3 |
| Live load | +112.1 | + 64.1 | + 31.7 | + 65.6 | — 50.8 | — 25.1 |
| Wind overturning: | | | | | | |
| On train from E | + 8.9 | + 5.4 | + 2.7 | + 4.3 | — 4.3 | — 2.1 |
| On train from W | — 8.9 | — 5.4 | — 2.7 | — 4.3 | + 4.3 | + 2.1 |
| Max. stress | +146.6 | + 78.0 | + 25.9 | + 78.9 | — 66.4 | — 24.9 |
| Min. stress | + 16.3 | 0 | 0 | + 9.0 | — 4.5 | — 4.5 |

To save space, the stresses are taken to the nearest 100 pounds, and are then all

expressed in thousand-pound units, + for tension, — for compression.

The minimum stress in B E is the same as that in B′ E′ or (Art. 57), — 8.414 from live load, + 25.555 from dead load and (Art. 65) — .892 from wind overturning (one panel C′ loaded) = 16.3.

| | Batter Brace | Upper Chord | | Lower Chord | | |
|---|---|---|---|---|---|---|
| | A B | B D | D F | A C | C E | E G |
| Dead load..... | — 42.9 | — 41.5 | — 46.7 | + 26.0 | + 26.0 | + 41.5 |
| Live load..... | —170.0 | —157.8 | —176.8 | +103.4 | +103.4 | +157.8 |
| Wind | | | | | | |
| on truss from E | ..... | 0 | + 5.6 | 0 | + 9.4 | + 15.0 |
| " " " W | ..... | — 5.6 | — 7.5 | — 9.4 | — 15.0 | — 16.9 |
| on train from E | ..... | ..... | ..... | 0 | + 18.8 | + 30.0 |
| " " " W | . ... | ..... | ..... | — 18.8 | — 30.0 | — 33.8 |
| Do. Overturn'g: | | | | | | |
| on truss from E | — 15.4 | ..... | ..... | + 9.4 | + 9.4 | + 9.4 |
| " " " W | + 15.4 | ..... | ..... | — 9.4 | — 9.4 | — 9.4 |
| on train from E | — 13.4 | — 13.1 | — 14.7 | + 8.2 | + 8.2 | + 13.1 |
| " " " W | + 13.4 | — 13.1 | + 14.7 | — 8.2 | — 8.2 | — 13.1 |
| Maxim. stress | —241.7 | —212.4 | —232.6 | +147.0 | +175.2 | +266.8 |
| Min. stress | — 27.5 | — 41.5 | — 41.1 | + 7.2 | + 1.2 | + 15.2 |

In computing maximum and minimum stresses from the tabular values, be careful

to combine only those stresses which must occur at the same time

Thus the maximum stress in D F is due to "dead and live load" (— 46.7 — 176.8), "wind overturning" on truss and train from east (— 14.7) and "wind on truss" from east (+ 5.6), giving in all a stress of (— 232.6).

In finding the minimum for D F no train is supposed on the bridge, and the wind from the east is seen to give the least stress = — 46.7 + 5.6 = — 41.1.

The stresses in the upper lateral bracing (Art. 63) are seen to be very small. In cases of this kind, the members are designed arbitrarily, no section being allowed less than three-fourths of a square inch. Stiff lateral bracing is of material aid in preventing vibration. In the lower system the floor beams usually act as the struts, the diagonals being suitably fastened to them.

In the last table above it will be noticed that the wind from the west, on the empty truss, comes near reversing the stress in

the second panel C E of the lower chord. In cases where this happens the panels where reversion can occur should be designed to act as struts as well as ties, though sometimes the stringers are relied on to take the exceptional compression.

Some engineers take no notice of the stresses due to wind in the vertical trusses when they are less than those due to live and dead-loads, unless they cause a reversal of stress in the lower chord, the reason being that such stresses are extremely unlikely to occur in the lifetime of the bridge, and that when they do occur they never strain the metal up to the elastic limit on account of the low unit stresses taken for ordinary dead and live-load stresses. Therefore, it is claimed that such exceptional stresses can be safely allowed a few times in the life of the bridge. Others again provide for these extra stresses due to wind when they exceed 25 per cent. of the stresses due to the dead and live-load combined, by specifying that the section of member shall be increased until the total stress (including wind) per square

inch shall not exceed by more than 25 per cent. the maximum fixed for live and dead load only. The members C E and E G are the only ones of this truss that fall under the last rule.

The stresses in the end (batter) posts are increased over those given above by the action of the wind transferred to the tops of the end posts by the upper lateral bracing (Art. 63), which, through the medium of the portal bracing, causes additional stresses in the end posts from the bending moments induced. Lack of space prevents a discussion of portal bracing here. Certain forms of portal bracing were discussed by the author in Engineering News for July 13, 1889, but a very full analysis will be found in *Johnson's Framed Structures*, to which the reader is referred.

67. *Determination of Theoretical Sectional Areas.*—For finding the sectional areas of the members of the vertical truss the following formulas* for unit stress in

* See *Transactions Am. Soc. C. E.* for June, 1886, and for Feb. 1892, for discussions on formulas for unit stresses. Specifications differ very much with regard to unit stresses.

thousand pounds per square inch for steel will be used:

$$\text{For ties, } a = 10\ (1 + \theta)$$

$$\text{For posts, } b = a\left(1 - \frac{1}{220}\,\frac{l}{r}\right)$$

In these formulas,

$\theta$ = Minimum stress on member / Maximum stress on member

$l$ = length of member in inches

$r$ = least radius of gyration of member in inches.

| Member | Min. Stress | Max. Stress | $\theta$ | $a$ | $\frac{l}{r}$ | Unit Stress | Area |
|---|---|---|---|---|---|---|---|
| A C E | 26.0 | 129.4 | .20 | 12.0 | | 12.0 | 10.8 |
| E G | 41.5 | 199.3 | .21 | 12.1 | | 12.1 | 16.5 |
| B E | 17.1 | 137.7 | .123 | 10.1 | | 10.1 | 13.6 |
| D G | 0 | 72.6 | 0 | 10.0 | | 10.0 | 7.3 |
| E F | 0 | 23.2 | 0 | 10.0 | | 10.0 | 2.3 |
| B C | 9.0 | 74.6 | .12 | 11.2 | | 11.2 | 6.7 |
| D E | 4.5 | 62.1 | .07 | 10.1 | 90 | 6.0 | 10.4 |
| F G | 4.5 | 22.9 | .20 | 12.0 | 125 | 5.2 | 4.4 |
| A B | 42.9 | 212.8 | .20 | 12.0 | 70 | 8.2 | 25.9 |
| B D | 41.5 | 199.3 | .208 | 12.1 | 43 | 9.7 | 20.5 |
| D F | 46,7 | 223.5 | .209 | 12.1 | 43 | 9.7 | 23.1 |

The maximum and minimum stresses are those due to live and dead loads only,

and are taken from Art. 57. The values of $\frac{l}{r}$ are assumed.

We first find $\theta$ by dividing min. stress by max. stress; then $a = 10\ (1 + \theta)$, giving the "unit stress" for ties. The "unit stress" $b$ for columns is then found for the assumed values of $l \div r$ by the formula above. On dividing the "max. stress" by "unit stress," the "area" of the section of member in square inches is found. Commercial sizes of "shapes" must be used. This will alter the net section of the ties very little; of the posts and upper chords more. The sections of the latter, though, are often very much altered to satisfy the conditions of assembling the parts on the pins, particularly as a minimum thickness of metal is always specified.

In fact, it may be well to conclude with the remark that the work has only fairly begun when the stresses in a bridge have been computed and "designing" has been reached, of which only the simplest elements have been given above.

## APPENDIX.

MOST ECONOMICAL HEIGHT OF TRUSSES WITH PARALLEL CHORDS.—The weight of the material of a truss that varies with its height $h$ is a function of $h$; denote it by F ($h$). This weight, for chords and web, can be found by multiplying the theoretical section (Art. 67) of each member by its length in feet multiplied by a constant, and summing the products for the entire truss. The other parts of the truss (pins, rivets, cover plates, floor system, transverse bracing, etc.) do not vary perceptibly with $h$; hence F ($h$) is the weight of material computed on chord and web stresses only, and it is a minimum when

$$\frac{d\,\mathrm{F}\,(h)}{d\,h} = \text{line}\frac{\mathrm{F}\,(h+\triangle h)-\mathrm{F}\,(h)}{\triangle h} = o \ldots . \quad (1)$$

Denote the weight of both chords that varies with $h$ by $W_c$; also denote the variable part of the weight of a web member by $w$, and call its inclination to the vertical $i$ and its length $l$. When the height of the truss is changed to $h + \triangle h$ (fig. 29), call the new value of $l$, $l + \triangle l$.

The formulas for unit stresses per square inch are (see Art. 67 for meaning of $\theta$ and $r$):

Fig. 29.

For ties, $a = u\,(1 + \theta)$..... .....(2)

For columns, $b = a\left(1 - c\,\frac{l}{r}\right)$ .....(3)

If we suppose the panel-length and unit stresses in the chords to remain invariable, since the arms of the chord members are $h$ and $h + \triangle h$ before and after the change in height, the new stresses in them are $h \div (h + \triangle h)$ of the stresses before the increase in height, and hence the new weight of the variable material in the chord is

$$W'_c = W_c\,\frac{h}{h + \triangle h} = W_c - W_c\,\frac{\triangle h}{h + \triangle h} \quad \ldots (4)$$

If S denotes the shear on a web strut the old stress is S *sec i* and its volume $\frac{S\ sec\ i}{b}\,l = \frac{S\,l^2}{b\,h}$; similarly the new volume is $\frac{S\,(l + \triangle l)^2}{b\,(h + \triangle h}$, where $b$ is given by *e q*, (3) and $b'$ by the same formula, when $l$ is changed to $(l + \triangle l)$. For ties, $b$ and $b'$ are replaced by $a$.

The new weight $w'$ of the variable material in a web member is thus equal to $w$ multiplied by the ratio of new to old volumes, or,

$$w' = w\,\frac{(l + \triangle l)^2}{l^2}\ \ \frac{h}{h + \triangle h}\ \frac{b}{b'}\ , \quad \ldots\ldots\ldots\ldots (5)$$

For ties, the ratio $b \div b'$ is to be replaced by 1.

Actually dividing the value of $b$ given by (3) by the value of $b'$, we find, regarding $r$ as constant,

$$\frac{b}{b'} = \frac{1 - c\frac{l}{r}}{1 - c\frac{l + \triangle l}{r}} = 1 + \frac{\frac{c}{r} \triangle l}{1 - \frac{c}{r}(l + \triangle l)}$$

If we place $k = \dfrac{\frac{c}{r}}{1 - \frac{c}{r}(l + \triangle l)}$, this ratio can be written, $\dfrac{b}{b'} = 1 + k \triangle l$.....(6)

where $k$ for ties $= 0$. On substituting (6) in (5),

$$w' = w\frac{(l + \triangle l)^2}{l^2}\frac{h}{h + \triangle h} + w\frac{(l + \triangle l)^2}{l^2}\frac{hk \triangle l}{h + \triangle h}$$

$$= w + w\frac{2hl \triangle l + h \triangle l^2 - l^2 \triangle h}{l^2 h + l^2 \triangle h}$$

$$+ w\frac{(l + \triangle l)^2}{l^2}\frac{h k \triangle l}{h + \triangle h} \quad \text{.....(7)}$$

The sum of the new weights of all the web members will be designated by $\Sigma(w')$, the sum of the old by $\Sigma(w)$,

Therefore, as $F(h) = W_c + \Sigma(w)$

$$F(h + \triangle h) = W'c + \Sigma(w')$$

$$\therefore \frac{F(h + \triangle h) - F(h)}{\triangle h} = -\frac{W_c}{h + \triangle h} + \frac{\Sigma(w') - \Sigma w}{\triangle h}$$ from eq. (4)

Hence, writing out $\Sigma(w') - \Sigma(w)$, dividing by $\triangle h$ and noting that $lim\ \dfrac{\triangle l}{\triangle h} = cos\ i$, as $\triangle h$

and $\triangle\ l$ diminish indefinitely, we have for a minimum weight of truss by (1), on taking limits :

$$\frac{-\ W_c}{h} + \Sigma\left(\frac{2\,h\cos i - l}{l\,h} + \cos i \times \lim k\right) w = o.$$

Since $\cos i = h \div l$, the above becomes on multiplying through by $h$,

$$W_c = \Sigma\left[\frac{2\,h^2}{l^2} - 1 + \frac{h^2}{l^2}\, l\,(\lim\ k)\right] w$$

Now, $\frac{2\,h^2}{l^2} - 1 = 2\cos^2 i - 1 = \cos 2\,i$, and placing $l\,(\lim k) = m$,

$$\therefore m = \frac{c\,\frac{l}{r}}{1 - c\,\frac{l}{r}} \quad \ldots\ldots\ldots (8)$$

we have finally,

$$W_c = \Sigma\, w\,(\cos 2\,i + m\,\cos^2 i) \ldots . \quad (9)$$

where $m$ is given by (8) for struts and $m = o$ for ties.*

*If, therefore, we multiply the variable weight of each web member by (cos 2 i + m cos² i) corresponding to it, the sum of the products should equal the weight of the two chords, if the most economical height has been chosen.*

For vertical members, $i = o$, $\cos i = \cos 2\,i = 1$. When $i < 45°$, $\cos 2\,i$ is $+$; $i = 45°$, $\cos 2\,i = 0$; $i > 45°$, $\cos 2\,i$ is minus.

* This formula being true for a Queen Post truss is likewise true when the middle panel reduces to zero, or for an A truss through or deck.

In Van Nostrand's Magazine for January, 1877, Emil Adler, C. E., deduced the formula $W_c = \Sigma (w \cos 2 i)$ for the most economical depth when $b$ is constant.

To apply (9), since the weight of any member is equal to its length in feet $\times$ cross-section in inches $\times$ $e$, where $e$ is the same for all members when they are of the same material, make out the weight of each member regarding $e$ as unity and substitute numerical results for the web members in a table headed as follows :

| Member | $w$ | $i$ | $\cos 2 i + m \cos^2 i$ | Product |
|---|---|---|---|---|
| | | | | |

The product $w (\cos 2 i + m \cos^2 i)$ is entered in the last column, and the sum of such products for all the web members, should by (9) equal $W_c$, the weight of the two chords, if the most economical height has been chosen. If $W_c$ is too large, then the height of truss may be increased, and vice versa.

The method is more nearly correct for long spans, where a larger number of members proportionately have sections near the theoretical sections. Some interesting conclusions can be drawn from (9).

(*a*) In a Warren girder, if $i = 45°$, $\cos 2i = o$ and $\Sigma (w m \cos^2 i)$ is quite small, so that for (9) to hold, $W_c$ must be small or the height of the truss large, which does not obtain for usual panel-lengths. It it thus absurd to have the web members inclined 45° if the least weight of truss is desired.

(*b*) In whipple or other multiple systems, $\cos 2i$ is nearer zero than for single intersection trusses of the same number of panels, hence the depth of truss is greater for the multiple systems.

(*c*) As a rule, angle $i$ should be less than 45°.

(*d*) The greater the number of panels for the same span, the heavier the web for the same kind of truss; hence by (9), $W_c$ must be greater or the truss lower for economy.

(*e*) For continuous girders it is known that $W_c$ is less than for a simple girder of the same span having the same weight of web; hence, if a certain height of truss satisfies the equality (9) for the simple girder, a less height is required for economy for the continuous girder.

(*f*) If the web in a deck bridge is heavier than the web in a through bridge of the same design, the height of truss must be less.

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